Prof. Friedrich Eisenbrand
Location: MA A3 31

Question session: 30.09.09
Discussion: 7.10.09

Exercises

Optimization Methods in Finance

Fall 2009

Sheet 1

Exercise 1.1
Write the following linear program in standard form

$$\begin{aligned} \max 4x_1 + x_2 - x_3 & \\ x_1 + 3x_3 &\leq 6 \\ 3x_1 + x_2 + 3x_3 &\geq 9 \\ x_1, x_2 &\geq 0 \\ x_3 &\in \mathbb{R} \end{aligned}$$

Exercise 1.2
Draw the feasible region (set of feasible solutions) of the following linear program (with 2 variables)

$$\begin{aligned} \max 2x_1 - x_2 & \\ x_1 + x_2 &\geq 1 \\ x_1 - x_2 &\leq 0 \\ 3x_1 + x_2 &\leq 6 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Determine the optimal solution to this problem by inspecting your drawing.

Exercise 1.3

i) For

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 3 & -1 & 4 \end{pmatrix}$$

compute a $d \in \mathbb{R}^3, d \neq \mathbf{0}$ with $Ad = \mathbf{0}$.

ii) Invert the matrix

$$B = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 3 & 0 \\ 3 & 4 & 1 \end{pmatrix}$$

iii) Compute a solution $x \in \mathbb{R}^3$ for $Bx = b$ with $b = (3,1,-1)^T$.

Exercise 1.4

The vector $x^* = (0,1,1,1)$ is an optimal solution of

$$\begin{aligned} \min\ (1,1,0,2) \cdot x & \\ \underbrace{\begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 2 \\ 2 & 0 & 0 & 0 \end{pmatrix}}_{=A} x &= \begin{pmatrix} 6 \\ 3 \\ 0 \end{pmatrix} \\ x &\geq \mathbf{0} \end{aligned}$$

with $x = (x_1, x_2, x_3, x_4) \in \mathbb{R}^4$. Use the proof of Lemma 2.1 to find another optimal solution x' such that $A_{J'}$ has full column rank with $J' = \{i \mid x'_i > 0\}$.

Exercise 1.5 – Practical exercise (2 points)

For the first practical exercise do the following (see the lecture notes for more details):

1. Transform the cashflow LP from the lecture into standard form.
2. Implement the naive linear programming algorithm to find an optimum solution (you can use the Boost C++-library).
3. Send the code together with compile instructions to `thomas.rothvoss@epfl.ch` until the 30th of September.