PhD Doctoral Course – Network Design – 15th September 2009

1st Assignment

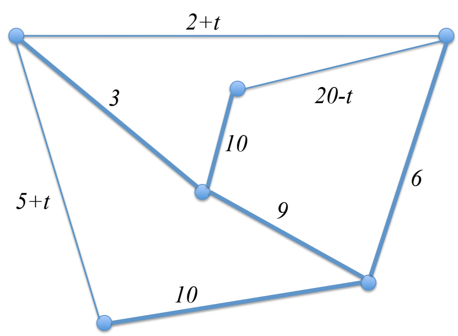
1. Show that the *Connector problem* and the *MST problem* are not equivalent if the edge costs are not assumed to be positive.

2. Show that any MST problem can be reduced to an MST problem with positive edge costs.

3. Prove that if $T(V,F)$ is an MST, and $e \in F$, then there is a cut induced by some set $D \subseteq V$ with $e \in \delta(D)$ and $c_e = \min\{c_f : f \in \delta(D)\}$.

4. Prove that a spanning tree $T(V,F)$ of $G(V,E)$ is an MST if and only if for every $e = \{uv\} \in (E \setminus F)$ and every edge f of an $u - v$ path in T, $c_e \geq c_f$.

5. For which values of t the bold tree in the picture is the *unique* MST for the given graph?



6. Show that the following algorithm finds an MST of a connected graph G. Begin with $H = G$. At each step, find (if one exists) a maximum cost edge e such that $H \setminus e$ is connected, and delete e from H.

7. Suppose that, instead of the *sum* of the costs of edges of a spanning tree, we wish to minimize the *maximum* cost of an edge of the spanning tree. That is, we want the most expensive edge of the tree to be as cheap as possible. This is called the *minmax spanning tree problem*. Prove that every MST actually solves this problem. Is the converse true?