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Discussions from: October 25, 2010

Combinatorial Optimization

Fall 2010

Assignment Sheet 3

Exercise 3

Reduce the problem of finding a maximum weight matching to the problem of finding a maximum weight perfect *matching. That is, find a way to transform, in polynomial time, any graph G with edge weights w into a graph G' with edge weights w' so that given a maximum weight perfect matching M' of G', one can easily deduce a maximum weight matching M of G.*

A simple way to achieve this is to give every vertex a "buddy" that it can be matched to if it remains unmatched by a matching in G. To be more precise, let $G = (V, E)$ with $V = \{v_1, \dots, v_n\}$. Then we define $G' = (V', E')$ with $2n$ vertices $V' = \{v_1, \dots, v_n, v'_1, \dots, v'_n\}$ and

$$E' = E \cup \{v'_i v'_j \mid v_i v_j \in E\} \cup \{v_i v'_i \mid i = 1 \dots n\}$$

We keep the weights of edges in E unchanged. The weight of all other edges is zero.

For every matching $M \subseteq E$ there exists a natural perfect matching $M' \subseteq E'$ of the same weight:

$$M' = M \cup \{v'_i v'_j \mid v_i v_j \in M\} \cup \{v_i v'_i \mid v_i \text{ not matched in } M\}$$

Conversely, for every perfect matching $M' \subseteq E'$, we can simply define $M = M' \cap E$ as a natural corresponding matching in G with the same weight.

So the maximum weight of a matching in G is equal to the maximum weight of a perfect matching in G', and morever, given a maximum weight perfect matching in G' we can easily obtain a maximum weight matching in G.