The problem can be submitted until April 5, 12 :00 noon, either at the exercise session or into the box in front of MA C1 563.

Student(s) [1] :

Question 1 : *The question is worth 5 points.*

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 *Reserved for the corrector*

Given a graph $G = (V, E)$ with a weight function $w : V \to \mathbb{R}$ on its vertices.
Consider the following linear program and its dual.

Primal

$$\begin{array}{rcll} \min \sum_{v \in V} w(v) x_v & & & \\ x_u + x_v & \geq & 1 & \forall \{u,v\} \in E \\ x_v & \geq & 0 & \forall v \in V \end{array}$$

Dual

$$\begin{array}{rcll} \max \sum_{e \in E} y_e & & & \\ \sum_{e \in E, e \ni v} y_e & \leq & w(v) & \forall v \in V \\ y_e & \geq & 0 & \forall e \in E \end{array}$$

Define $C_y \subseteq V$ to be the set of vertices for which the corresponding dual constraints are tight in y, i.e. $C_y = \{v \in V : \sum_{e \in E, e \ni v} y_e = w(v)\}$.
We apply the following algorithm for the dual linear program :

- Initialize the dual solution y to be $y_e = 0$ for every $e \in E$
- While there exists $\{u, v\} \in E$ such that $C \cap \{u, v\} = \emptyset$:
  Increase $y_{u,v}$ until one of the dual constraints (corresponding to u or v) becomes tight.
- Return y

Let y be the solution given by the algorithm and C_y its corresponding set of vertices. Show that for an optimal solution x^* of the primal linear program :

$$\sum_{v \in C_y} w(v) \leq 2 \sum_{v \in V} w(v) x_v$$

1. You are allowed to submit your solutions in groups of at most three students.