Discrete Optimization 2024 (EPFL): Problem set of week 13

May 24, 2024

1. Consider the polyhedron P defined by $Ax \leq b$ for

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ -1 & 1 & -2 \\ 3 & 1 & -1 \\ 2 & 2 & 1 \end{pmatrix}$$

   and $b = (1, 2, 3, 4, 5)$.

   Find a hyperplane separating P and $x = (1, 2, 3)$.

2. Find the volume of the cross polytope, the polytope with the $2n$ vertices in $\mathbb{R}^n$ that are $\pm e_1, \ldots, \pm e_n$.

3. Let P be the simplex with $n + 1$ integer vertices in $\mathbb{R}^n$. Assume that P contains an integer interior point. Prove that the volume of P is at least $\frac{n+1}{n!}$.

4. Let T be a non-singular linear transformation. Assume that the entries in the matrix representing T are at most D in absolute value and they are all integers. By at most how much can T shrink the size of a vector? That is, how small can be $\frac{|Tx|}{|x|}$? Give a meaningful lower bound.