

# Discrete Optimization 2024 (EPFL): Problem set of week 12

May 22, 2024

1. Verify that  $\{(x, y) \in \mathbb{R}^2 \mid 5x^2 - 2xy + 2y^2 \leq 1\}$  is an ellipse and find its area.
2. Let  $B$  be the unit ball in  $\mathbb{R}^3$  and let  $H$  be the hyperplane  $H = \{x + 3y + 2z = 0\}$ .

Consider the linear transformation

$$T = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

What is the (two dimensional) area of  $T(B \cap H)$ ?

3. Let  $E \subset \mathbb{R}^3$  be the ellipsoid  $E = \{(x, y, z) \mid \frac{(x+2y-z)^2}{4} + \frac{(y-x-z)^2}{9} + \frac{(2x-y-2z)^2}{25} \leq 2\}$ . What is the volume of  $E$ ?
4. Let  $E \subset \mathbb{R}^3$  be the ellipsoid  $E = \{(x, y, z) \mid x^2 + \frac{y^2}{4} + \frac{z^2}{9} \leq 1\}$ .  
Let  $H^+$  be the half-space  $H^+ = \{(x, y, z) \mid x + y + z \geq 0\}$ .  
Find an ellipsoid  $E'$  such that  $E' \supset E \cap H^+$  and  $Vol(E') \leq Vol(E)e^{-1/(2(3+1))}$ .