# Are large cuts necessary?

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## 1 Gomory-Chvátal cutting planes

Let $P = \{x \in \mathbb{R}^n \colon Ax \leq b\}$ be a polyhedron where $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$. For any $c \in \mathbb{Z}^n$ and $\delta \geq \max\{c^T x \colon Ax \leq b\}$, the inequality $c^T x \leq \lfloor \delta \rfloor$ is called a *Gomory-Chvátal cutting plane* [4, 1], or briefly *cutting plane* of $P$. It is valid for all integer points $x \in P \cap \mathbb{Z}^n$ and therefore also for the integer hull $P_I$ of $P$.

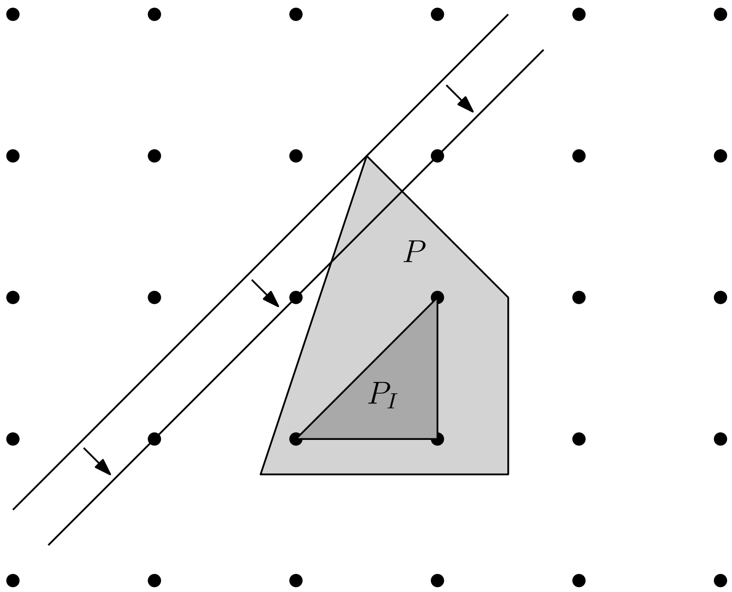


Figure 1: The valid inequality $-x_1 + x_2 \leq \delta$ yields the cutting plane $-x_1 + x_2 \leq \lfloor \delta \rfloor$.

For the ease of notation we will also write $c^T x \leq \delta$ for the set of points $\{x \in \mathbb{R}^n : c^T x \leq \delta\}$. The intersection of all cutting planes of $P$ is denoted by

$$P' = \bigcap_{\substack{(c^T x \leq \delta) \supseteq P \\ c \in \mathbb{Z}^n}} \left(c^T x \leq \lfloor \delta \rfloor\right). \tag{1}$$

By applying the closure operator $i$-times successively, one obtains $P^{(i)}$, the *$i$-th elementary closure* of $P \subseteq \mathbb{R}^n$.

The *Chvátal rank* [6] of a polyhedron $P$ is the smallest natural number $i \in \mathbb{Z}_{\geq 0}$ such that $P^{(i)} = P_I$. Schrijver [6] has shown that the Chvátal rank of a rational polyhedron is always finite.

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**The research problem**

In principle, one could generate cutting planes $c^T x \leq \lfloor \delta \rfloor$ such that $\|c\|_\infty$ is larger than the corresponding value of the largest facet normal-vector.

In this project, the student has to find out whether this is really necessary and eventually change the cutting-plane procedure such that large cutting planes never appear.

# References


[1] V. Chvátal. Edmonds polytopes and a hierarchy of combinatorial problems. *Discrete Mathematics*, 4:305–337, 1973.

[2] W. Cook, C. R. Coullard, and G. Turán. On the complexity of cutting plane proofs. *Discrete Applied Mathematics*, 18:25–38, 1987.

[3] W. Cook, A. M. H. Gerards, A. Schrijver, and É. Tardos. Sensitivity theorems in integer linear programming. *Math. Programming*, 34(3):251–264, 1986.

[4] R. E. Gomory. Outline of an algorithm for integer solutions to linear programs. *Bulletin of the American Mathematical Society*, 64:275–278, 1958.

[5] V. Reis and T. Rothvoss. The subspace flatness conjecture and faster integer programming. In *2023 IEEE 64th Annual Symposium on Foundations of Computer Science (FOCS)*, pages 974–988. IEEE, 2023.

[6] A. Schrijver. On cutting planes. *Annals of Discrete Mathematics*, 9:291–296, 1980.

[7] A. Schrijver. *Theory of Linear and Integer Programming*. John Wiley, 1986.