Learning for Adaptive and Reactive Robot Control
Solutions for theoretical exercises of lecture 1

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1.1 Optimal trajectories formulation

Book correspondence: Ex1.1, p8

Given a 2-joint planar robotic arm with unit link length, we are interested in its end-effector position along X direction, i.e. with forward kinematics:

$$x(t) = \cos(q_1(t)) + \cos(q_2(t)) = h(\mathbf{q}(t))$$

A trajectory is a path with $N+1$ waypoints $\{(q_1(0), q_2(0)), (q_1(1), q_2(1)), \ldots, (q_1(N), q_2(N))\}$, and a time-indexing $\{0, t(1), \ldots, t(N)\}$ that specifies the time at which the robot arrives the corresponding waypoint.

Write the optimisation problem to get the trajectories with $N+1$ waypoints and uniform time indexing, i.e. $\delta t = t(n) - t(n-1) = t(N)/N, \forall\, n \in [1, \ldots, N]$ that moves the robot from initial configuration x_0 given by $(q_1(0), q_2(0))$ to a desired position x^* subject to joint maximum increment $\delta q_{\max}$ per second with following desired properties:

(a) Minimizing the time to reach x^*

(b) Following the shortest path in Cartesian space

(c) Following the shortest path in joint space

Solution

We give the solution to K-joint robot directly. The three desired trajectory objectives stated in the exercise correspond to different objective functions for the optimisation problem. Hence, we begin by deriving the generic optimisation problem. Given the definition of a cost function $C(\mathbf{q}(i))$ which should be minimized at each time-step, the optimal path wrt. $C(\cdot)$ can be obtained by solving the following optimisation problem:

$$
\begin{aligned}
&\min_{\mathbf{q}(1),\ldots\mathbf{q}(N),T_f} \quad C(\cdot)\\
&\text{subject to}\\
&\quad\begin{cases}
\mathbf{x}(0) = \mathbf{x}_0,\\
\mathbf{x}(N) = \mathbf{x}^*,\\
\mathbf{x}(n) = h(\mathbf{q}(n)),\ ,\forall n \in [0\ldots N]\\
\dfrac{|q_i(n) - q_i(n-1)|}{\delta t} \leq \delta q_{max}, \forall i \in [1\ldots K],\ \forall n \in [0\ldots N]\\
N\delta t = T_f
\end{cases}
\end{aligned}
$$

where T_f corresponds to the time taken to reach the target. Next, we list the cost functions for the different objectives stated in the exercise description:

(a) To minimize the time taken to reach the target, we can define the cost function as:

$$C(\cdot) = T_f$$

indicating that we want T_f, which is the time taken to reach the target, to be minimal.

(b) To follow the shortest path in Cartesian space, we can use the following cost function:

$$C(\cdot) = \sum_{n=1}^{N} \|\delta\mathbf{x}(n)\|_2$$

By the triangle inequality, we can set that $\sum_{n=1}^{N} \|\delta\mathbf{x}(n)\|_2 \geq \|\sum_{n=1}^{N} \delta\mathbf{x}(n)\|_2 = \|\mathbf{x}^* - \mathbf{x}_0\|_2$. The solution is the line between the two points.

(c) To follow the shortest path in joint space, we can use the following cost function:

$$C(\cdot) = \sum_{n=1}^{N} \|\delta\mathbf{q}(n)\|_2$$

The same reasoning applies and the shortest path is a straight line in Joint space.

References

[1] Aude Billard, Sina Mirrazavi, and Nadia Figueroa. *Learning for Adaptive and Reactive Robot Control: A Dynamical Systems Approach.* MIT press, 2022.