Learning for Adaptive and Reactive Robot Control
Solutions for exercises of lecture 10

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Introduction

INTRO
This part of the course follows *exercises* 11.1 *and* 11.2 and *programming exercise* 11.1 of the book "*Learning for Adaptive and Reactive Robot Control: A Dynamical Systems Approach. MIT Press, 2022*".

1 Theoretical exercises [1h]

1.1

Book correspondence: Ex10.10
Let the nominal task model $f(x)$ be composed of conservative and non-conservative parts:

$$f(x) = f_c(x) + f_r(x)$$

Let the system $M(x)\ddot{x} + C(x,\dot{x})\dot{x} + g(x) = \tau_c + \tau_e$ be controlled by the following:

$$\tau_c = g(x) - D\dot{x} + \lambda_1 f_c(x) + \beta_R(z,s)\lambda_1 f_r(x)$$

where $z = \dot{x}^T f_r(x)$
The storage variable s has the following dynamics

$$\dot{s} = \alpha(s)\dot{x}^T D(x)\dot{x} - \beta_s(s,z)\lambda_1 z$$

and the following properties are satisfied,

$$\begin{aligned}
0 \le \alpha(s) \le 1 &\qquad s < \bar{s} \\
\alpha(s) = 0 &\qquad s > \bar{s} \\
\beta_s(z,s) = 0 &\qquad s \le 0 \;\; and \;\; z \ge 0 \\
\beta_s(z,s) = 0 &\qquad s \ge \bar{s} \;\; and \;\; z \le 0 \\
0 \le \beta_s(z,s) \le 1 &\qquad elsewhere \\
\beta_R(z,s) = \beta_s(z,s) &\qquad z \ge 0 \\
\beta_R(z,s) \ge \beta_s(z,s) &\qquad z < 0
\end{aligned}$$

Consider the storage function $W(x, \dot{x}, s) = \frac{1}{2}\dot{x}^T \mathrm{M}\dot{x} + \lambda_1 V_{\mathbf{c}(x)} + s$, where $V_c(x)$ is the potential function associated with $\mathrm{f}_c(x)$

Prove that if $0 < s(0) \leq s$, the resulting closed loop system is passive with respect to the input-output pair $\tau_e, \dot{x}$.

Solution:
First, note that $0 < s(0) \leq \overline{s} \Rightarrow 0 \leq s(t) \leq \overline{s}, \forall t > t_0$. Consider the storage function $W(x, \dot{x}, s) = \frac{1}{2}\dot{x}^T \mathrm{M}\dot{x} + \lambda_1 V_{\mathbf{c}(x)} + s$, where $V_c(x)$ is the potential function associated with $\mathrm{f}_c(x)$. The rate of change of W is:

$$\dot{W}(x, \dot{x}) = x^T \mathrm{M}\ddot{x} + \frac{1}{2}\dot{x}^T \dot{M}\dot{x} + \lambda_1 \nabla V_c^T \dot{x} + \dot{s} \tag{1}$$

Substituting $\mathrm{M}\ddot{x}$ and τ_c using the skew-symmetry of $\dot{\mathrm{M}} - 2\mathrm{C}$ yields:

$$\begin{aligned} \dot{W}(x, \dot{x}) = &- \dot{x}^T \mathrm{D}\dot{x} + \dot{x}^T \tau_e + \beta_R(z, s)\lambda_1 z + \\ &+ \lambda_1 \dot{x}^T \mathrm{f}_c(x) + \lambda_1 \nabla V_c^T \dot{x} + \dot{s} \end{aligned} \tag{2}$$

The second-to-last two terms cancel because $\mathrm{f}_c(x) = -\nabla V_c(x)$. Substituting $\dot{s}$ then yields:

$$\dot{W}(x, \dot{x}) = -\underbrace{(1 - \alpha(s))}_{\geq 0} \dot{x}^T \mathrm{D}\dot{x} + \zeta(z, s)\lambda_1 z + \dot{x}^T \tau_e \tag{3}$$

where $\zeta(z, s) = \beta_R(z, s) - \beta_s(z, s)$ has been introduced to ease the notation. We have $1 - \alpha(s) \geq 0$ and $\zeta(z, s) = 0$ for all $z > 0$ and $\zeta(z, s) \geq 0$ for $z < 0$. Hence, we have:

$$\dot{W}(x, \dot{x}) \leq \dot{x}^T \tau_e \tag{4}$$

which concludes the proof.