Learning for Adaptive and Reactive Robot Control
Solutions for exercises of lecture 3

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1 Exercise 1 - Dynamical Systems and Stability

1.1 Exercise 1.1

Consider a 2 dimensional linear DS, $\dot{x} = Ax$, with $x(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. We wish to introduce a modulation matrix M to modify the dynamics as follows: $\dot{x} = MAx$. Given $M = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$,

1. Find a diagonal matrix $A = \mathbf{diag}(a_1, a_2)$, with $a_1 \neq a_2$ for which the system converges to $x^* = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$
2. Compute the path integral of the modulated DS

Solution

1. We only need to find a matrix A such that the eigenvalues of MA are with negative real parts. Let $A = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}$, then eigenvalues of MA are (a_1, a_2). Therefore, any diagonal matrix A with negative diagonal elements will make the system converge to $x^* = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

2. Let $A = \begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix}$, then the corresponding DS is $\dot{x} = MAx = \begin{pmatrix} -2 & 2 \\ 0 & -1 \end{pmatrix} x$, where MA can be diagonalized as,

$$\begin{pmatrix} -2 & 2 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix} = PDP^{-1}$$

then according to Slides 19, the path integral can be written as,

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = Pe^{(Dt)}P^{-1}x(0)$$

References

[1] Aude Billard, Sina Mirrazavi, and Nadia Figueroa. *Learning for Adaptive and Reactive Robot Control: A Dynamical Systems Approach*. MIT press, 2022.