Learning for Adaptive and Reactive Robot Control
Instructions for exercises of lecture 4

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Introduction

INTRO

This part of the course follows *exercises* 3.2, 3.3 *and* 3.5 and *programming exercises* 3.1 *to* 3.4 of the book "*Learning for Adaptive and Reactive Robot Control: A Dynamical Systems Approach. MIT Press, 2022*".

1 Theoretical exercises [1h]

1.1

Design a matrix $A \in \mathbb{R}^{2\times 2}$ and Lyapunov function shaping matrix $P \in \mathbb{R}^{2\times 2}$ to ensure that a linear dynamical system (DS),

$$\dot{x} = f(x) = A(x - x^*)$$

with another attractor at the origin $x^* = [0\ 0]^T$ to be globally, asymptotically stable (GAS) with respect to the conditions stated using either:

(a) A matrix $Q \in \mathbb{R}^{2\times 2}$ with the following form:

$$Q = q\,\mathbb{I}_2, \quad q \in \mathbb{R}$$

Solution: Remember that, the system is GAS if

$$A^T P + PA = Q, \quad P = P^T \succ 0, \quad Q = Q^T \prec 0$$

with matrix A and P given as:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad P = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix},$$

The Q matrix is evaluated as

$$Q = \begin{bmatrix} 2a_{11}p_{11} + 2a_{21}p_{12} & a_{11}p_{12} + a_{12}p_{11} + a_{21}p_{22} + a_{22}p_{12} \\ a_{11}p_{12} + a_{12}p_{11} + a_{21}p_{22} + a_{22}p_{12} & 2a_{12}p_{12} + 2a_{22}p_{22} \end{bmatrix} \tag{1}$$

The diagonal Q matrix is negative definite if $q < 0$. The following choice of values can achieve this:

$$p_{11} > 0,\ p_{22} = p_{11}\frac{a_{11}}{a_{22}},\ p_{12} = 0, \quad \text{and} \quad a_{11} < 0,\ a_{22} < 0,\ a_{12} = (-1)a_{21}\frac{p_{11}}{p_{22}}$$

We obtain:

$$Q = \begin{bmatrix} 2a_{11}p_{11} & 0 \\ 0 & 2a_{11}p_{11} \end{bmatrix}$$

(b) *Optional* A matrix $Q \in \mathbb{R}^{2\times 2}$ with the following form:

$$Q = \begin{bmatrix} q_1 & q_2 \\ q_2 & q_1 \end{bmatrix}, \quad q_1, q_2 \in \mathbb{R}$$

Solution: The matrix Q is negative definite if the eigenvalues are smaller than zero, i.e.,

$$\lambda_{1,2} = q_1 \pm q_2 < 0 \quad \Rightarrow \quad q_1 < 0,\ |q_2| > |q_1|$$

We propose similar values, but only soften the constraints for a_{12}, a_{21}:

$$|a_{12}p_{11} + a_{21}p_{22}| > 2a_{11}p_{11}$$