Learning for Adaptive and Reactive Robot Control
Instructions for exercises of lecture 6

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Introduction

INTRO
This part of the course follows *exercises* 8.1 *to* 8.6 and *programming exercises* 8.1 *to* 8.6 of the book "*Learning for Adaptive and Reactive Robot Control: A Dynamical Systems Approach. MIT Press, 2022*".

1 Theoretical exercises [1h]

1.1

Consider the nominal DS $\dot{x} = Ax$ with $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$. Construct a matrix $M(x)$ that is locally active and:

1. **Solution:** The final dynamics are given as

$$\dot{\hat{x}} = M(x)Ax$$

   The system is moving away from the attractor, if any eigenvalue of $(M(x)A)$ is greater than zero. A possible choice is

$$M(x) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

2. **Solution** The attracor becomes a saddle point if one of the eigenvalues is positive and one negative, for example,

$$M(x) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

3. create a limit cycle around the attractor, (a) keep the system stable at the attractor or (b) make the system diverge from the attractor and converge to the limit cycle (see Fig. 2)
   Solution: The limit cycle around the attractor with radius r can be formed by locally rotating the dynamical system.

$$M(x) = R(\phi) = \begin{bmatrix} \cos(\phi(x)) & -\sin(\phi(x)) \\ \sin(\phi(x)) & \cos(\phi(x)) \end{bmatrix}$$

to form a limit cycle, we need a rotation of at least $|\phi(x)| = \pi/2$, where π is the circle constant.
(a) to be stable at the attractor, we set:

$$\phi(x) = \begin{cases} \frac{\pi}{2}\sin\left(\|x\|\pi/r\right) & \text{if } \|x\| < r \\ 0 & \text{otherwise} \end{cases}$$

(b) to move away from the attractor and converge to the limit cycle we set:

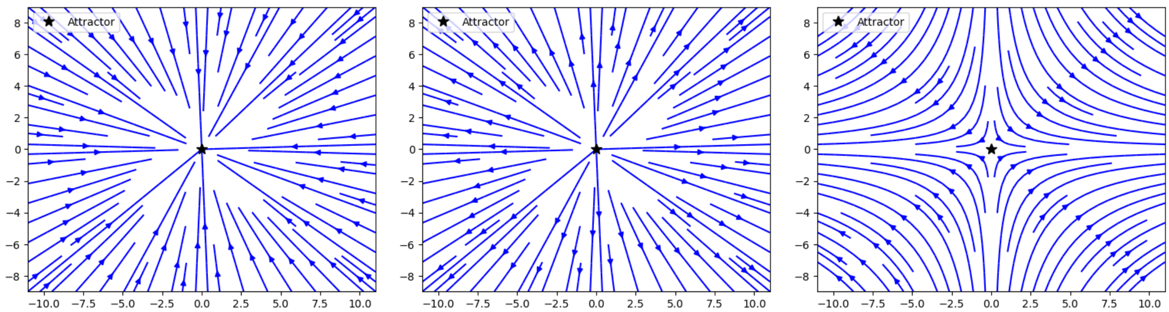
$$\phi(x) = \max\left(\pi\left(1 - \frac{\|x\|}{2r}\right), 0\right)$$

4. invert the direction of the initial dynamics at a fixed point x^*.
Make sure your modulation matrix is smooth.

The flow is inverted if we use a modulation matrix of the form:

$$M(x) = \begin{bmatrix} 1-\gamma & 0 \\ 0 & 1-\gamma \end{bmatrix} \quad \text{with } \gamma = c^1 \exp\left(-\frac{1}{\sigma^2}\|x - x^*\|\right)$$

where $c^1 = 4$. This forces the dynamical system to flip in the surrounding region.



(a) Initial dynamics (b) Unstable System (c) Semi-stable System

Figure 1: The different vector fields of the modulations.

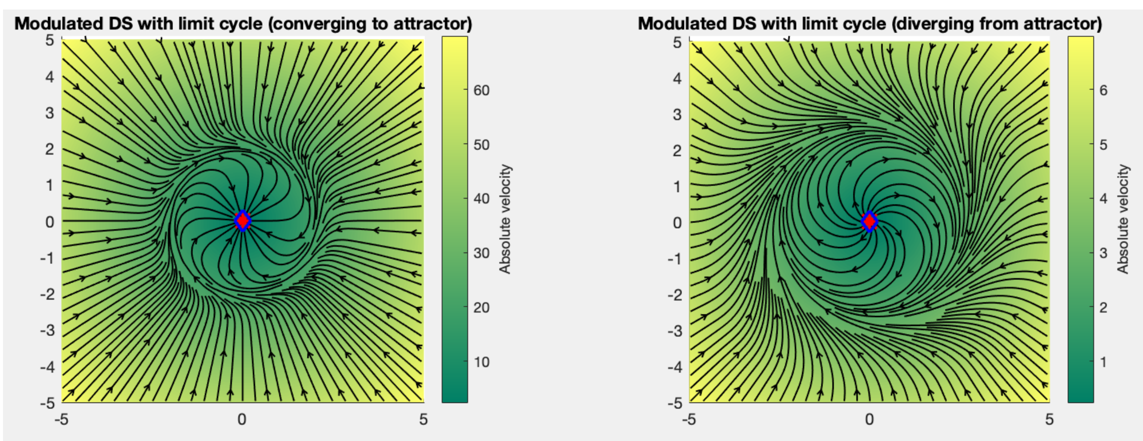


Figure 2: Two limit cycles at the attractor

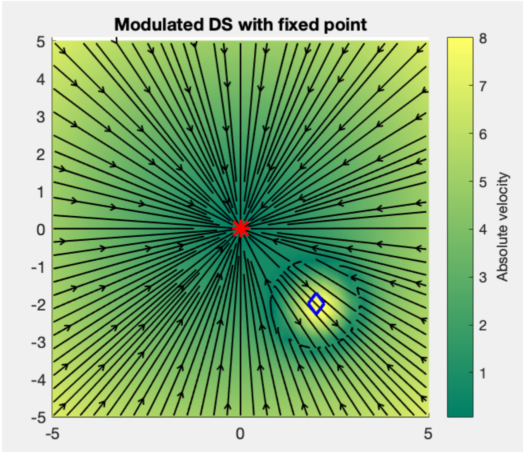


Figure 3: Locally inverted flow.

References

[1] Aude Billard, Sina Mirrazavi, and Nadia Figueroa. *Learning for Adaptive and Reactive Robot Control: A Dynamical Systems Approach.* MIT press, 2022.