Learning for Adaptive and Reactive Robot Control
Instructions for exercises of lecture 7

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Introduction

INTRO

This part of the course follows *exercises* 9.1 *to* 9.3 and *programming exercise* 9.1 of the book "*Learning for Adaptive and Reactive Robot Control: A Dynamical Systems Approach. MIT Press, 2022*".

1 Theoretical exercises [1h]

Consider the nominal DS $\mathbf{f}(x) = Ax + b$ with $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ and $b = [-1, 1]^T$. We then construct a modulation to add a circular obstacle as follows:

$$\dot{x} = \mathbf{M}(x)\mathbf{f}(x) \quad \text{with} \quad \mathbf{M}(x) = \mathbf{E}(x)\mathbf{D}(x)\mathbf{E}(x)^{-1}, \tag{1}$$

where $M(x)$ is build through eigenvalue decomposition, using as the basis of the eigenvectors the normal $n(x)$ and tangents to the obstacle :

$$\mathbf{E}(x) = \begin{bmatrix} \mathbf{n}(x) & \mathbf{e}_1(x) \end{bmatrix}, \qquad \mathbf{D}(x) = \begin{bmatrix} \lambda_n(x) & 0 \\ 0 & \lambda_e(x) \end{bmatrix},$$

where the tangent $\mathbf{e}(x)$ forms an orthonormal basis to the gradient of the distance function $d\Gamma(x)/dx$.

We then set the eigenvalues to cancel the flow in the normal direction once it reaches the boundary of the obstacle:

$$\lambda_n(x) = 1 - \frac{1}{\Gamma(x)} \quad \text{and} \quad \lambda_e(x) = 1 + \frac{1}{\Gamma(x)}$$

With $\Gamma(x)$ our distance function constructed so that :

- $\Gamma(x) > 1$ outside the obstacle
- $\Gamma(x) = 1$ at the obstacle boundary

- $\Gamma(x) < 1$ inside the obstacle

Since we want to model a circular obstacle, we first recall the circle equation :

$$d(x, x_o)^2 = r^2,$$

where x_o is the center of the circle, $d(x, x_o)$ is the euclidean distance between x and x_o and r is the radius of the circle. We can implicitly embed the circle equation into $\Gamma(x)$ to create a circular obstacle, while still keeping the properties cited above by defining $\Gamma(x)$ as :

$$\Gamma(x) = d(x, x_o)^2 - r^2 + 1.$$

This definition ensures $\Gamma(x)$ follows the prerequisites to implement obstacle avoidance in our DS.

1.1

The modulation generated by the eigenvalues results in a change of magnitude along the various basis directions, as described earlier. The increase in velocity along the tangent direction is bounded.

- What is its upper bound?
  Solution: The upper bound when the eigenvalue reaches the maximum. The highest eigenvalue we can obtain is in tangent direction, with $\lambda_e(x) = 2$. Hence, the maximum speed up is a factor of 2.

- Where does it occur?
  Solution: It occurs on the surface of the obstacle, i.e. and the velocity being parallel to the tangent, i.e.

$$\dot{x} = \lambda_e(x)\mathbf{f}(x) = 2\,\mathbf{f}(x) \qquad \frac{\dot{x}^T\mathbf{f}(x)}{\|x\|\,\|\mathbf{f}(x)\|} = 1,\, \Gamma(x) \approx 1$$