Learning for Adaptive and Reactive Robot Control
Instructions for exercises of lecture 9

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Introduction

INTRO

This part of the course follows *exercises* 10.1 *to* 10.8 and *programming exercises* 10.1 *to* 10.3 of the book "*Learning for Adaptive and Reactive Robot Control: A Dynamical Systems Approach. MIT Press, 2022*".

1 Theoretical exercises [1h]

1.1

Book correspondence: Ex10.1, p272

Consider a second-order, one-dimensional (1D) DS, which can be used to hit a target at $x = x^*$, with desired velocity $\dot{x} = \dot{x}^*$ at $t = t^*$. Consider first the undamped system for the DS using equation:

$$m\ddot{x} + k(x - x^*) = 0, \qquad x(0) = x_0$$

and then consider a DS following equation:

$$m\ddot{x} + d(\dot{x} - \dot{x}^*) + k(x - x^*) = 0, \qquad x(0) = x_0$$

Is the damping term required?
What if the DS needs to pass through two points in space with two different velocities (i.e., $x = x_1^*$ with $\dot{x} = \dot{x}_1^*$ at $t = t_1^*$ and $x = x_2^*$ with $\dot{x} = \dot{x}_2^*$ at $t = t_2^*$). Can this scenario be accomplished? Support your answer with analytical and numerical results.
Solution:

The analytical solution of a spring mass system is

$$x(t) = c_1 \sin\left(\sqrt{\frac{k}{m}}t + \Delta\phi\right) \tag{1}$$

$$\dot{x}(t) = \sqrt{\frac{k}{m}}c_1 \cos\left(\sqrt{\frac{k}{m}}t + \Delta\phi\right) \tag{2}$$

Even though there are two unknown parameters in the equation, m, k, their ratio defines the behaviour of the system, Hence, in general the damping term is needed to hit the object at the desired time, desired location with the desired velocity.

Same applies to the two points scenario. In general, it is not possible to pass two points at the specific time with specific velocity.

References

[1] Aude Billard, Sina Mirrazavi, and Nadia Figueroa. *Learning for Adaptive and Reactive Robot Control: A Dynamical Systems Approach*. MIT press, 2022.