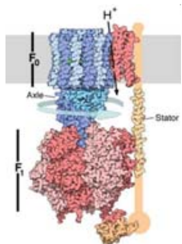
12: Energie des rotations et le moment cinétique

I. Quelle est l'énergie mécanique d'un solide en rotation ?
  Rappel: Dynamique
II. Comment déterminer le moment cinétique d'un solide ?
  Rotations déséquilibrées - centrifugeuse
III. La loi de la conservation du moment cinétique sert à quoi ?



ATP synthase

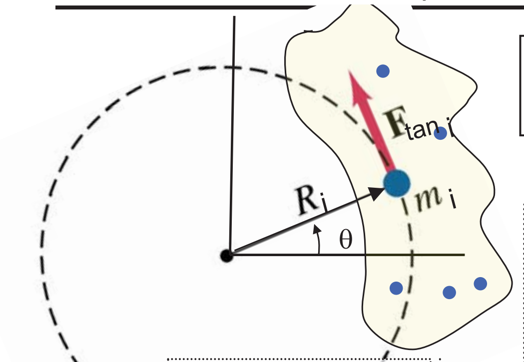
Préparation au cours et aux exos

Chapitres du Giancoli à lire **avant le cours** *(1.5 p)*:
  11-1 Angular Momentum – objects rotating about a fixed axis
Exercices simples *(6)* à faire **avant la séance d'exos:**
  Giancoli 10-65, 71
  11-2, 4, 6, 15

Giancoli chapitres 10-8, 10-9; 11-4 à 11-6

Rappel: La 2ème loi de Newton des rotations

est une conséquence directe de la 2ème loi linéaire



2ème loi (linéaire):

$\mathbf{F}_{tan,i} = m_i \mathbf{a}_{tan,i}$

$$\tau_i = F_i R_i$$

$$a_{T,i} = \frac{dv_i}{dt} = R_i \frac{d\omega_i}{dt}$$

$$\tau_i = m_i R_i^2 \frac{d\omega_i}{dt}$$

$$= m_i R_i^2 \alpha$$

Objet indéformable:
$\omega_i = \omega \rightarrow d\omega/dt = \alpha$

Sommation (intégration):

$$\sum_{\lim m_i \to 0} m_i R_i^2 = \int R^2 dm \equiv I$$

La différence:
couple τ
et moment d'inertie I
sont définis pr à un axe

$$\sum \vec{F} = M\vec{a}_{CM}$$

$$\sum \vec{\tau}_{ext}^{\,axe} = I^{axe} \vec{\alpha}$$

(voir leçons 5, 10 et 11)

(Presque) toute la dynamique en bref

force et couple, masse et moment d'inertie, énergie cinétique

NB. Les lois de rotation sont une conséquence directe des lois de Newton du mouvement linéaire. Les lois sont alors jumelées …

	linéaire (voir leçons 4,5,7, & 10)	rotation (voir leçons 5, 9 -12)
Une dimension (composante)	$\sum F = ma$	$\sum \tau_{ext}^{axe} = I^{axe}\alpha$
vectorielle	$\sum \vec{F} = M\vec{a}_{CM}$	$\sum \vec{\tau}_{ext}^{axe} = I^{axe}\vec{\alpha}$ $\vec{\tau} \equiv \vec{r} \times \vec{F}$
Masse, moment d'inertie	$M = \int dm$ (Résistance à changer la vitesse)	$I^{axe} = \int_{\text{autour axe de rotation}} r^2 dm$ (Résistance à changer la vitesse angulaire)
Energie cinétique	$K = M\frac{v_{CM}^2}{2}$	$K_{rot} = I\frac{\omega^2}{2}$

12-1. Quelle est l'énergie mécanique de rotation ?

(rotation autour d'un axe fixe)

La cinétique et dynamique des rotations sont directement dérivées de la dynamique linéaire:

⇒ expression de l'énergie des rotations équivalent à celle de l'énergie cinétique:

$$K_{rot} = \frac{I^{axe}\omega^2}{2}$$

$$I^{axe} = \sum(\Delta m r^2) = \int_{objet} r^2 dm$$

L'énergie mécanique

E_{tot}= K+U+K_{rot}

pour un objet de masse M avec une rotation autour de son CM:

$$E_{tot} \equiv M\vec{g}\cdot\vec{r}_{CM} + M\frac{v_{CM}^2}{2} + I^{CM}\frac{\omega^2}{2}$$

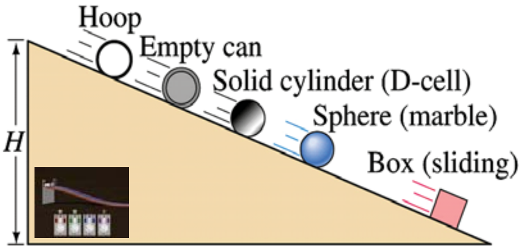
U_{CM} K_{CM} K_{rot}

Conservation de l'énergie mécanique

Si, ΣF_{ext}=0 et $\Sigma\tau_{ext}$=0, pour un système **conservatif** en rotation, l'énergie mécanique est conservée :

E = K_{CM}+K_{rot}+U_{CM} = constante

Course des cylindres : par conservation d'énergie



Alors, qui arrive le premier en bas?

	$\frac{I^{CM}}{MR^2}$	$\frac{K_{CM}}{MgH}$
cyl.creux	1	1/2
cyl.plein	1/2	2/3
sphère	2/5	5/7
boîte		1

$$MgH = M\frac{v_{CM}^2}{2} + K_{rot}$$

Transformation de K_{rot} $\quad K_{rot} = I^{CM}\frac{\omega^2}{2}$

$$K_{rot} = \left(\frac{I^{CM}}{MR^2}\right)M\frac{v_{CM}^2}{2}$$

$$\omega = v_{CM}/R$$

$$MgH = M\frac{v_{CM}^2}{2}\left(1+\frac{I^{CM}}{MR^2}\right) \longrightarrow \overbrace{M\frac{v_{CM}^2}{2}}^{K_{CM}} = MgH\frac{1}{\left(1+\frac{I^{CM}}{MR^2}\right)}$$

Quels phénomènes/quantités physiques observez-vous ?

C E R F S P A R E I L

Collisions (in) élastiques - quantité de mouvement

Equilibre stable sur table - Energie potentielle constante

Roulement des boules

Frottement statique

Stabilisation gyroscopique de la table

P - Quantité de mouvement

A - Accélération angulaire

R - Référentiel accélérée

I - et d'inertie

E - Energie mécanique

L - Moment cinétique

12-2. Le moment cinétique

2ème loi de Newton du point matériel (leçons 8-9)

$\vec{p} \equiv m\vec{v}$ $\qquad \sum \vec{F}_i = \frac{d\vec{p}}{dt}$

$\vec{L}^0 \equiv \vec{r} \times \vec{p} = m r_\perp^2 \vec{\omega}$ $\qquad \sum \vec{\tau}^O{}_i = \frac{d\vec{L}^O}{dt}$

Sans forces extérieures **nettes**:

Conservation de p

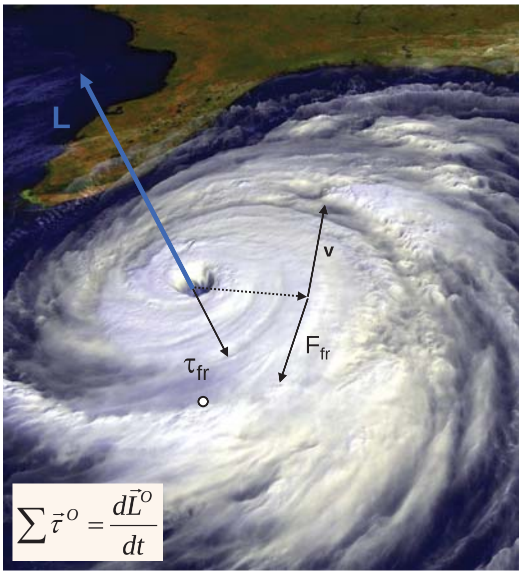
Sans moments de force **nets**:
Conservation de L

NB. $\vec{r} \times \vec{p} = \vec{r}_\perp \times \vec{p} = m\vec{r}_\perp \times (\vec{\omega} \times \vec{r}_\perp) = m r_\perp^2 \vec{\omega}$

	Mouvement linéaire (voir leçons 4, 8 - 10)	Mouvement de rotation (voir leçons 5, 9-11)
1D	$F_{ext} = Ma_{CM} \quad (a_{CM} = dv_{CM}/dt)$ $\left(\sum F_i = Ma_{CM}\right)$	$\tau_z = I\alpha_z \quad (\alpha_z = d\omega_z / dt)$ $\left(\sum \tau_z = I\alpha\right)$
vectorielle	$\vec{F}_{ext} = \frac{d\vec{p}_{CM}}{dt} \quad (\vec{p} = M\vec{v}_{CM})$	$\sum \vec{\tau}^O = \frac{d\vec{L}^O}{dt} \qquad \vec{L}^O \equiv I^O \vec{\omega}$

$M = \int dm$ $\qquad I^{axe} = \int_{\text{autour axe}} r^2 dm$

Les Ouragans



Constats: Les Ouragans se forment sur l'eau chaude et sont maintenus en rotation avec des vents puissants de 200-300km/h pendant des jours.

L'air en rotation est soumis à des forces de frottement avec l'air qui a une vitesse plus petite et aussi avec le contact de l'eau.

Le résultat est un couple qui sert à diminuer le moment cinétique …

Question I: Quelle est la force/énergie qui soutient un ouragan ?

Question II: Que peut-on dire des conséquences du réchauffement climatique ?

http://webphysics.iupui.edu/webscience/physics_archive/hurricanes.html

http://www.physics.ubc.ca/outreach/phys420/p420_04/sean/

Quelles forces subit un axe équilibré sous rotation ?

Situation: Roue en rotation autour d'un axe fixe, maintenue par les roulements, eux-mêmes fixés solidement, e.g. à un châssis, ω_z=constante.

Modèle: Une barre (masse négligée) portant 2 masses m à R, fixée perpendiculairement à l'axe de la roue.

2ème loi rotations:

Moment cinétique pr. à O:

$$\vec{L}^O = \vec{L}_1^{\,O} + \vec{L}_2^{\,O} = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 \quad p = mv = m\omega R \quad (v = \omega R)$$

$$L_z^{\,O} = 2mR^2\omega_z = const \qquad L_x^{\,O} = L_y^{\,O} = 0$$

Moments de force (pr à O)

$$\vec{\tau}^{\,O} = \vec{0} \rightarrow z_A F_{Ax} + z_B F_{Bx} = 0$$

$$z_A = -z_B$$

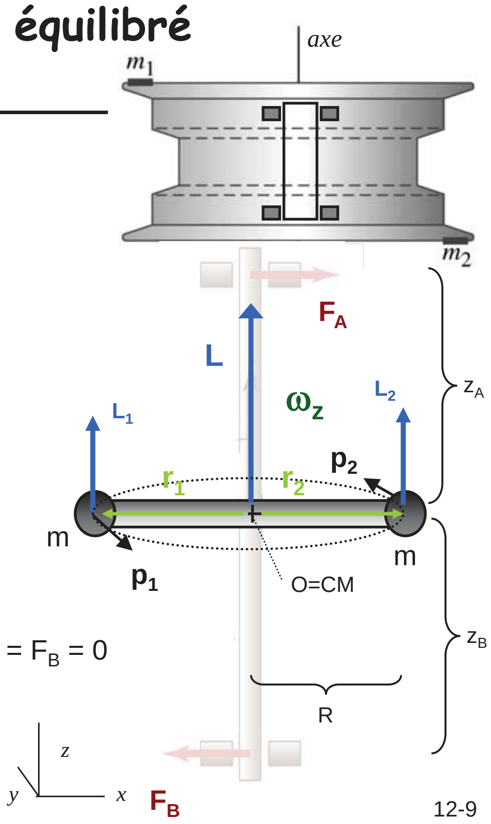
$$F_{Ax} = F_{Bx}$$

2ème loi linéaire : $\mathbf{a}_{CM}$=0

$$F_{Ax} + F_{Bx} = Ma_{CM}$$

$$\mathbf{F_{Ax}=-F_{Bx}}$$

$F_A = F_B = 0$



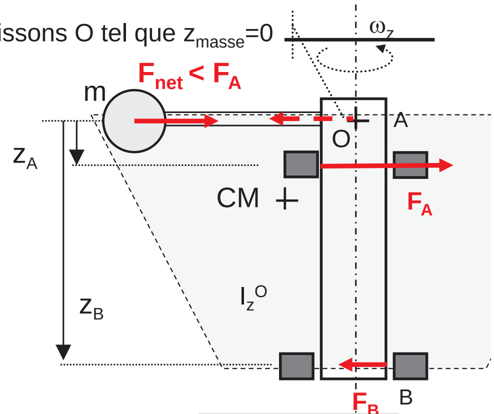
Constat: Pas de forces sur les roulements

Quelles forces la centrifugeuse déséquilibrée subit-elle ?

Conséquence de la diapo précédente: Une centrifugeuse équilibrée n'est pas soumise à un couple (analyse à un instant donné).

Situation : Ajout d'une masse m unique, placée en haut (leçon 5)

Choisissons O tel que z_{masse}=0



2ème loi rotation:

Moment cinétique pr. à O:

$$L_z^{\,O} = mR^2\omega_z + I_z^{\,O}\omega_z = const$$

Moments de force (pr à O)

$$\tau_y^{\,O} = z_A F_A + z_B F_B = 0$$

i) $$F_B = -\frac{z_A}{z_B} F_A$$

2ème loi linéaire :

1) Pour m (mouvement circulaire):
$F_{net}=F_R=m\omega^2R$

2) Pour l'axe (qui ne bouge pas, a=0):
$-F_{net}$ (reactio) = F_A+F_B

ii) $$F_A + F_B = m\omega^2 R$$

ii) - i) $$F_A = \frac{m\omega^2 R}{\left(1 - \frac{z_A}{z_B}\right)} > F_R$$

Exemple: masse de m = 1g, fréquence f de 20000rpm (ω~2000 rad/s), rayon R = 5 cm. Hauteur de la centrifugeuse: z_A = 2cm, z_B = 5 cm:

$F_A = F_R/(1-2/5) = 0.001\ 2000^2\ 0.05 / 0.6 = 200/0.6$

→ F_A = 330 N (poids de 33kg).

⇒ Nécessité d'équilibrer la charge d'une centrifugeuse en disposant symétriquement les éprouvettes.

12-3. Conservation du moment cinétique

(rotation autour d'un axe de direction fixe)

Constat:

1.La masse = résistance à changer la vitesse
$\Rightarrow$ Conservation de la quantité de mouvement p

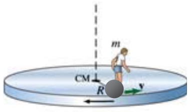
2.Le moment d'inertie = résistance à changer la vitesse angulaire

$$\sum_{i=1}^{N} \vec{\tau}^{O}_{i,net} = \frac{d\sum_{i=1}^{N} \vec{L}^{O}_{i}}{dt} = \boxed{\frac{d\vec{L}^{O}}{dt} = \sum \vec{\tau}_{O,net}}$$

Loi de conservation du moment cinétique:
Pour un système isolé ($\tau^{O}_{net}=0$)
$L_{tot}(t)=constante$

Situation: Rotation autour d'un axe de direction fixe.
Si le système est isolé: $\tau^{O}_{,net}=0$

$$\Rightarrow L^{O}_{,z} = I^{O}_{z}\omega = constante$$



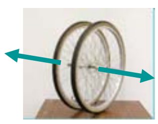
Quiz

Situation: Le démonstrateur se trouve initialement immobile sur un plateau capable de tourner sans frottement. On lui donne une roue de vélo en rotation autour d'un axe vertical.

Il change l'orientation de l'axe avec un angle de 180^{0}.

Question: Que se passe-t-il ?

A. rien, il reste immobile sur le plateau
B. Il se met à tourner dans le sens opposé à la rotation initiale de la roue
C. Il se met à tourner dans le même sens dans lequel tournait initialement la roue.



Quels conséquences de varier le moment d'inertie d'un système isolé ?

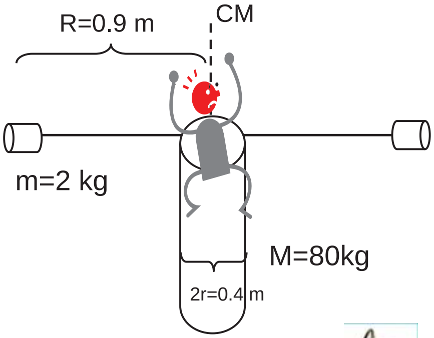
Exemple I: Un pizzaiolo fait de la pâte à pizza. La pâte s'étend, jusqu'à ce que la force qui maintienne la pâte en une pièce, modélisée comme force ressort –kr, satisfait la condition centripète $m\omega^2 r$.

Pourquoi faut-il réappliquer un couple τ?

Liu, K.-C.; Friend, J.; and Yeo, L. "The behavior of bouncing disks and pizza tossing." *Europhysics Letters*, 85 (2009) 60002, doi: 10.1209/0295-5075/85/60002.

Exemple II: On se trouve en rotation sur une chaise qui peut se tourner librement avec deux poids (2 kg) dans ses mains. On étend les bras.

Que se passe-t-il ?



Humain: $I^{CM}=Mr^2/2 = 1.6$ kg m^2

Masses: $I^{CM}=2mR^2 = 3.2$ kg m^2

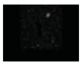
I^{CM} augmente au moins 3 fois !

Quand on raccourcit les bras :
I^{CM} diminue au moins 3 fois !

Ex. Pulsar du Crabe

étoile à neutrons

Le reste d'une supernova observée par des astronomes chinois et arabes en 1054-1056.



Aujourd'hui son rayonnement illumine le gaz éjecté : Nébuleuse du Crabe

Données:

Diamètre: 20 - 30km

Masse 1.3-2 M_S

Période de rotation T: 33ms (190 rad/s), augmente 35 ns/jour (0.4 ps/s)

Puissance ?

$$P = \frac{dK_{rot}}{dt} = I\omega\frac{d\omega}{dt} = \frac{2MR^2}{5}\frac{\omega^3}{2\pi}\frac{dT}{dt}$$

$= 3\cdot10^{30}$ [kg] $1.5\ 10^8$ [m^2] 190^3 $0.4\cdot10^{-12}/6.3$

$\sim 10^{32}$ W

Soleil:

$M_S = 2\cdot10^{30}$ kg

$R_S = 110R_T = 7\cdot10^5$ km

Période = 25 j

Supernova:

$$L = \frac{2}{5}M_S R^2\omega = \frac{4\pi}{5}M_S\frac{R^2}{T}$$

Système isolé, donc L conservé

Contraction à 30 km ($4\cdot10^{-5}R_S$): I diminue $5\cdot10^9$ fois

ω augmente 10^9 fois : période réduite 10^9 fois = 4ms

La conservation du moment cinétique

– à votre disposition !

fusils/canon

Apportez vos Powerballs pour cours #13!!

Déterminer si un œuf est cuit ou pas

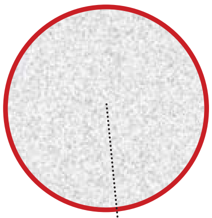
International space station

Système de navigation

Pointeur sans fil

Complément: Démonstration de l'énergie mécanique de rotation: K_{rot}

$v_{CM}=0$



$\mathbf{r}_i$ et $\mathbf{v}_i$ pr au repère en O (CM)

$v_i = \omega_z r_i$

$$K = \frac{1}{2}\sum_i m_i v_i^2 = \frac{\omega_z^2}{2}\underbrace{\sum_i m_i r_i^2}_{I_z^{CM}}$$

Solide indéformable

$$\Rightarrow \quad K_{rot} = \frac{1}{2} I_z^{CM} \omega_z^2$$

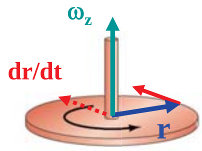
Si $v_{CM} \neq 0$ il faut ajouter l'énergie cinétique linéaire de l'objet :

$$K = \frac{1}{2} M v_{CM}^2 + \frac{1}{2} I_z^{CM} \omega_z^2$$

Complément: Rappel de la cinématique et dynamique des rotations

Cinématique angulaire

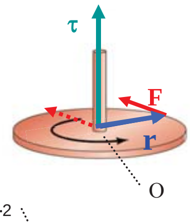
Vitesse angulaire instantanée : $\Rightarrow \frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$



Accélération angulaire instantanée

$$\vec{\alpha}(t) \equiv \frac{d\vec{\omega}(t)}{dt}$$

Dynamique des rotations



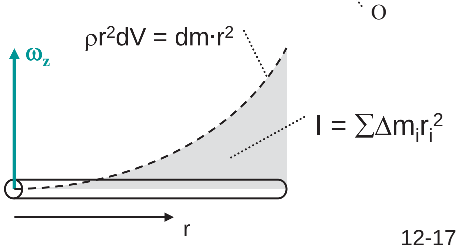
Le moment d'une force pr à un point O

$$\Rightarrow \quad \vec{\tau}_{\vec{r}_0} \equiv \Delta\vec{r} \times \vec{F} = (\vec{r} - \vec{r}_0) \times \vec{F}$$

Moment d'inertie autour d'un axe de rotation

$$I \equiv \int_{\substack{\text{autour axe} \\ \text{de rotation}}} r^2 dm$$

r = distance pr axe de rotation



Complément: Bobine et fil - analyse pr au CM

2e loi rotation :

$$\tau_z^{CM} = RF_f - rF = \frac{d\omega_z}{dt} I_z^{CM}$$

$$\frac{d\omega_z}{dt} \underset{\omega=-\frac{v_{CM}}{R}}{=} -\frac{a_{CM}}{R} \quad \text{(roulement)}$$

2ème loi de Newton (translation) :

$$F \cos\theta + F_f = Ma_{CM}$$

$$RF_f - rF = -\frac{a_{CM}}{R} I_z^{CM} \qquad -RF_f - RF \cos\theta = -MRa_{CM}$$

addition

$$-RF \cos\theta - rF = -MRa_{CM} - \frac{a_{CM}}{R} I_z^{CM}$$

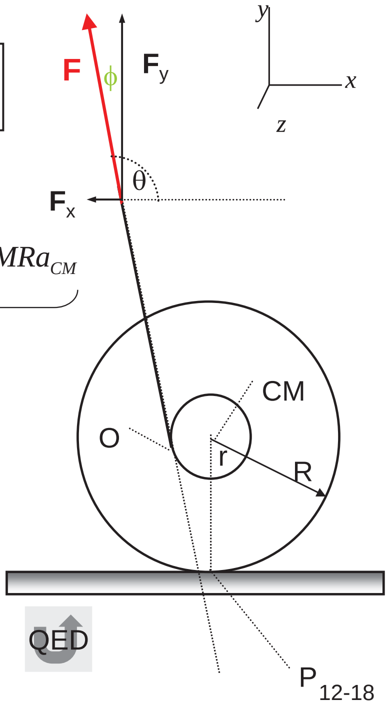
$$FR(\cos\theta + r/R) = M\left(R + \frac{I_z^{CM}}{MR}\right) a_{CM}$$

$$a_{CM} = \frac{F}{M} \frac{(r/R + \cos\theta)}{\left(1 + \frac{I_z^{CM}}{MR^2}\right)}$$

a_{CM} prend des valeurs positives ainsi que négatives selon θ:
Quand $a_{CM} = 0$: $R\cos\theta = -r \rightarrow R\sin\phi = r$

$\Rightarrow$ F est selon OP!

$\cos\theta = -\sin(\theta - 90) = -\sin\phi$



QED

11–1 Angular Momentum—Objects Rotating About a Fixed Axis

In Chapter 10 we saw that if we use the appropriate angular variables, the kinematic and dynamic equations for rotational motion are analogous to those for ordinary linear motion. In like manner, the linear momentum, $p = mv$, has a rotational analog. It is called **angular momentum**, L, and for an object rotating about a fixed axis with angular velocity ω, it is defined as

$$L = I\omega, \quad \textbf{(11–1)}$$

where I is the moment of inertia. The SI units for L are $kg \cdot m^2/s$; there is no special name for this unit.

We saw in Chapter 9 (Section 9–1) that Newton's second law can be written not only as $\Sigma F = ma$, but also more generally in terms of momentum (Eq. 9–2), $\Sigma F = dp/dt$. In a similar way, the rotational equivalent of Newton's second law, which we saw in Eqs. 10–14 and 10–15 can be written as $\Sigma\tau = I\alpha$, can also be written in terms of angular momentum: since the angular acceleration $\alpha = d\omega/dt$ (Eq. 10–3), then $I\alpha = I(d\omega/dt) = d(I\omega)/dt = dL/dt$, so

$$\Sigma\tau = \frac{dL}{dt}. \quad \textbf{(11–2)}$$

This derivation assumes that the moment of inertia, I, remains constant. However, Eq. 11–2 is valid even if the moment of inertia changes, and applies also to a system of objects rotating about a fixed axis where $\Sigma\tau$ is the net external torque (discussed in Section 11–4). Equation 11–2 is Newton's second law for rotational motion about a fixed axis, and is also valid for a moving object if its rotation is about an axis passing through its center of mass (as for Eq. 10–15).

Conservation of Angular Momentum

Angular momentum is an important concept in physics because, under certain conditions, it is a conserved quantity. What are the conditions for which it is conserved? From Eq. 11–2 we see immediately that if the net external torque $\Sigma\tau$ on an object (or system of objects) is zero, then

$$\frac{dL}{dt} = 0 \quad \text{and} \quad L = I\omega = \text{constant.} \qquad [\Sigma\tau = 0]$$

This, then, is the **law of conservation of angular momentum** for a rotating object:

The total angular momentum of a rotating object remains constant if the net external torque acting on it is zero.

The law of conservation of angular momentum is one of the great conservation laws of physics, along with those for energy and linear momentum.

When there is zero net torque acting on an object, and the object is rotating about a fixed axis or about an axis through its center of mass whose direction doesn't change, we can write

$$I\omega = I_0\omega_0 = \text{constant.}$$

I_0 and ω_0 are the moment of inertia and angular velocity, respectively, about the axis at some initial time ($t = 0$), and I and ω are their values at some other time. The parts of the object may alter their positions relative to one another, so that I changes. But then ω changes as well and the product $I\omega$ remains constant.

Many interesting phenomena can be understood on the basis of conservation of angular momentum. Consider a skater doing a spin on the tips of her skates, Fig. 11–1. She rotates at a relatively low speed when her arms are outstretched, but when she brings her arms in close to her body, she suddenly spins much faster. From the definition of moment of inertia, $I = \Sigma mR^2$, it is clear that when she pulls her arms in closer to the axis of rotation, R is reduced for the arms so her moment of inertia is reduced. Since the angular momentum $I\omega$ remains constant (we ignore the small torque due to friction), if I decreases, then the angular velocity ω must increase. If the skater reduces her moment of inertia by a factor of 2, she will then rotate with twice the angular velocity.

A similar example is the diver shown in Fig. 11–2. The push as she leaves the board gives her an initial angular momentum about her center of mass. When she curls herself into the tuck position, she rotates quickly one or more times. She then stretches out again, increasing her moment of inertia which reduces the angular velocity to a small value, and then she enters the water. The change in moment of inertia from the straight position to the tuck position can be a factor of as much as $3\frac{1}{2}$.

Note that for angular momentum to be conserved, the net torque must be zero, but the net force does not necessarily have to be zero. The net force on the diver in Fig. 11–2, for example, is not zero (gravity is acting), but the net torque about her CM is zero because the force of gravity acts at her center of mass.

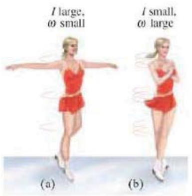
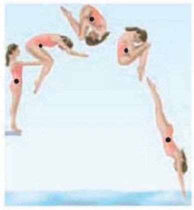


FIGURE 11–1 A skater doing a spin on ice, illustrating conservation of angular momentum: (a) I is large and ω is small; (b) I is smaller so ω is larger.

FIGURE 11–2 A diver rotates faster when arms and legs are tucked in than when they are outstretched. Angular momentum is conserved.



65. (II) A merry-go-round has a mass of 1640 kg and a radius of 7.50 m. How much net work is required to accelerate it from rest to a rotation rate of 1.00 revolution per 8.00 s? Assume it is a solid cylinder.

71. (I) A bowling ball of mass 7.3 kg and radius 9.0 cm rolls without slipping down a lane at 3.7 m/s. Calculate its total kinetic energy.

Directional Nature of Angular Momentum

Angular momentum is a vector, as we shall discuss later in this Chapter. For now we consider the simple case of an object rotating about a fixed axis, and the direction of $\vec{\mathbf{L}}$ is specified by a plus or minus sign, just as we did for one-dimensional linear motion in Chapter 2.

For a symmetrical object rotating about a symmetry axis (such as a cylinder or wheel), the direction of the angular momentum† can be taken as the direction of the angular velocity $\vec{\omega}$. That is,

$$\vec{\mathbf{L}} = I\vec{\omega}.$$

As a simple example, consider a person standing at rest on a circular platform capable of rotating friction-free about an axis through its center (that is, a simplified merry-go-round). If the person now starts to walk along the edge of the platform, Fig. 11–5a, the platform starts rotating in the opposite direction. Why? One explanation is that the person's foot exerts a force on the platform. Another explanation (and this is the most useful analysis here) is as an example of the conservation of angular momentum. If the person starts walking counterclockwise, the person's angular momentum will be pointed upward along the axis of rotation (remember how we defined the direction of $\vec{\omega}$ using the right-hand rule in Section 10–2). The magnitude of the person's angular momentum will be $L = I\omega = (mR^2)(v/R)$, where v is the person's speed (relative to the Earth, not the platform), R is his distance from the rotation axis, m is his mass, and mR^2 is his moment of inertia if we consider him a particle (mass concentrated at one point). The platform rotates in the opposite direction, so its angular momentum points downward. If the initial total angular momentum was zero (person and platform at rest), it will remain zero after the person starts walking. That is, the upward angular momentum of the person just balances the oppositely directed downward angular momentum of the platform (Fig. 11–5b), so the total vector angular momentum remains zero. Even though the person exerts a force (and torque) on the platform, the platform exerts an equal and opposite torque on the person. So the net torque on the *system* of person plus platform is zero (ignoring friction) and the total angular momentum remains constant.

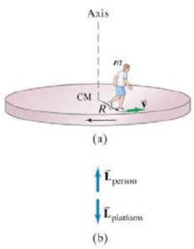


FIGURE 11–5 (a) A person on a circular platform, both initially at rest, begins walking along the edge at speed v. The platform, assumed to be mounted on friction-free bearings, begins rotating in the opposite direction, so that the total angular momentum remains zero, as shown in (b).

2. (I) (*a*) What is the angular momentum of a 2.8-kg uniform cylindrical grinding wheel of radius 18 cm when rotating at 1300 rpm? (*b*) How much torque is required to stop it in 6.0 s?

4. (II) A figure skater can increase her spin rotation rate from an initial rate of 1.0 rev every 1.5 s to a final rate of 2.5 rev/s. If her initial moment of inertia was $4.6 \, kg \cdot m^2$, what is her final moment of inertia? How does she physically accomplish this change?

6. (II) A uniform horizontal rod of mass M and length ℓ rotates with angular velocity ω about a vertical axis through its center. Attached to each end of the rod is a small mass m. Determine the angular momentum of the system about the axis.

15. (II) A nonrotating cylindrical disk of moment of inertia I is dropped onto an identical disk rotating at angular speed ω. Assuming no external torques, what is the final common angular speed of the two disks?