Bernoulli - Abstracts

October 7, 2024

Contents

1 David Wallauch-Hajdin

Title: On optimal blowup stability for wave maps
Abstract: This talk reports on an optimal blowup stability result for the wave maps equation, which was obtained jointly with R. Donninger. More precisely, we establish the stability of an explicitly known self-similar profile under co-rotational perturbations which are small in the critical L^2-based Sobolev space. The main tool used to prove this result is a set of new Strichartz estimates in similarity variables.

2 Akansha Sanwal

Title: Well-posedness for dispersion generalised KP-I equations
Abstract: The KP-I equation models waves which propagate essentially in one dimension with weak transverse effects. We consider the dispersion generalised version of the KP-I equation on ^{2} and $\times$ and prove new well-posedness results in anisotropic Sobolev spaces. We obtain the sharp dispersion rate below which these equations exhibit quasilinear behaviour. In the quasilinear regime, we use short-time Fourier restriction to prove well-posedness results. Nonlinear Loomis-Whitney inequalities and bilinear Strichartz estimates are the main ingredients of the proof. The talk is based on joint works with Shinya Kinoshita and Robert Schippa.

3 Bjoern Bringmann

Title: Invariant Gibbs measures for $(1 + 1)$-dimensional wave maps into Lie groups.
Abstract: We consider the wave maps equation for maps from (1+1)-dimensional Minkowski space into a compact Lie group. The Gibbs measure of this model corresponds to a Brownian motion on the Lie group, which is a natural object from stochastic differential geometry. Our main result is the invariance of the Gibbs measure under the wave maps equation and is the first result of this kind for any geometric wave equation. The proof combines techniques from differential geometry, partial differential equations, and probability theory.

4 Charles Collot

Title: On patterns of singularity formation for the parabolic-elliptic Keller-Segel system
Abstract: The parabolic-elliptic Keller-Segel system models cell motion under chemotaxis. It is a mass preserving equation that has the same scaling invariance as the quadratic semilinear heat equation. It admits blowup solutions in the mass critical two-dimensional case as well as in higher dimensions that are mass supercritical. In such instances when the density becomes singular in finite time, this describes cell aggregation. This talk will first review four previously known blow-up patterns (self-similar, flat, collapsing steady state, collapsing sphere). It will then present a new one in the mass critical case: where two stationary states are simultaneously collapsing and colliding at a single singular point. A formal blow-up law was proposed by Herrero-Seki-Velazquez in 2014. We provide a rigorous construction of such solution. We will explain some of the new ideas to study this dynamics that to our knowledge had not been studied before in evolution pdes, where two solitons interact in the same parabolic neighborhood from the singularity in a non-radial configuration, together with the radiation remainder. This is joint work with T.-E. Ghoul (NYU Abu Dhabi), N. Masmoudi (NYU Abu Dhabi and Courant Institute) and V. T. Nguyen (National Taiwan University).

5 Frédéric Valet

Title: Orbital stability of a traveling wave of the Gross-Pitaevskii equation
Abstract: The Gross-Pitaevskii (GP) equation in dimension 2 is a model for Bose-Einstein condensates. The solutions have a condition at infinity : the modulus of the solutions goes to 1 at infinity, which makes the analysis different from the usual non-linear Schrödinger equation. Unlike the 1-dimensional case, it is also not kwown if this non-linear dispersive equation is integrable and

the momentum is defined only formally. Due to the competition between the dispersion and the non-linearity, there exist traveling waves moving at a velocity $c \in (0, \sqrt{2})$. In this talk, we investigate a new proof of the orbital stability of traveling waves with small speed using an adequate “quadratic” form. More precisely, we explain how to obtain a suitable definition of the momentum and detail how the non-linearity strongly influences the ”quadratic” form at infinity. The talk is based on a collaboration with Philippe Gravejat and Eliot Pacherie (CY Cergy Paris Université).

6 Irfan Glogic

Title: On self-similar blowup for supercritical dispersive PDEs
Abstract: Numerical simulations of many types of supercritical evolution equations indicate that both the generic singularity formation and threshold for blowup phenomena are driven by self-similar solutions.
In this talk, we explore these observations for the focusing cubic wave equation in the energy supercritical case, $d \geq 5$. We first survey the results leading to a complete proof of non-radial stability of the trivial ODE blowup profile. Then, we exhibit what appears to be the only nontrivial self-similar solution that is known in closed form. We then follow with numerical evidence that this solution is a generic attractor within the threshold for ODE blowup. Finally, as the first step toward rigorously showing this observation, we outline our proof of non-radial co-dimension one stability of this solution. This is joint work with Birgit Schörkhuber.
At the end, if time permits, we will comment on the analogous results/conjectures for other supercritical dispersive models.

7 Jacek Jendrej

Title: Statistical mechanics of the wave maps equation in dimension 1+1
Abstract: We study wave maps with domain $\mathbb{R}^{1+1}$ and values in $\mathbb{S}^d$, defined on the future light cone $\{|x| \leq t\}$, with prescribed data at the boundary $\{|x| = t\}$. Based on the work of Keel and Tao, we prove that the problem is well-posed for locally absolutely continuous boundary data. We design a discrete version of the problem and prove that for every absolutely continuous boundary data, the sequence of solutions of the discretised problem converges to the corresponding continuous wave map as the mesh size tends to 0. Next, we consider the boundary data given by the $\mathbb{S}^d$-valued Brownian motion. We prove that the sequence of solutions of the discretised problems has an accumulation point for the topology of locally uniform convergence. We argue that the resulting random field can be interpreted as the wave-map evolution corresponding to the initial data given by the Gibbs distribution. This is a joint work with Zdzisław Brzeźniak.

8 James Coe

Title: Sharp quasi-invariance threshold for the cubic Szegő equation
Abstract: The cubic Szegő equation serves as a toy model for weakly-dispersive nonlinear Hamiltonian dynamics. We shall consider the flow of a family of Gaussian fields under these dynamics. We see that above a critical regularity, the measures are quasi-invariant under the flow, but below this regularity, quasi-invariance fails. In fact, the distribution at almost every future time is singular with respect to the initial distribution. We will discuss a general heuristic of when we expect quasi-invariance or singularity in Hamiltonian systems and test this hypothesis for the Szegő dynamics. In the high-regularity regime we prove standard energy estimates, and in the low-regularity setting we exhibit an instantaneous growth of Sobolev norms of the solution (at high frequencies), and then employ an abstract argument to show that such a growth cannot occur with positive probability at all times. This talk is based on joint work with Leonardo Tolomeo.

9 Leonardo Tolomeo

Title: Statistical mechanics of the focusing nonlinear Schrödinger equation
Abstract: In this talk, we discuss a number of results related to (non-)construction of the Gibbs measures for focusing nonlinear Schrödinger equations.
The program was initiated by Lebowitz-Rose-Speer (1988), who built the focusing Φ^p measure in dimension 1 by introducing a suitable mass cutoff. A number of open questions were raised in this work, namely the existence of certain phase transitions and if the construction can be repeated in higher dimension.
In this talk, we show the solution to most of those questions. After surveying various results, we will discuss the general strategy for tackling these problems, and the main techniques developed to find the answer.
This talk is based on joint works with T. Oh (Edinburgh), M. Okamoto (Osaka), H. Weber (Münster) and J. Forlano (Edinburgh).

10 Louise Gassot

Title: Zero-dispersion limit for the Benjamin-Ono Equation
Abstract: We focus on the Benjamin-Ono equation on the line with a small dispersion parameter. The goal of this talk is to precisely describe the solution at all times when the dispersion parameter is small enough. This solution may exhibit locally rapid oscillations, which are a manifestation of a dispersive shock. The description involves the multivalued solution of the underlying inviscid Burgers equation, obtained by using the method of characteristics. This work is in collaboration with Elliot Blackstone, Patrick Gérard and Peter D Miller.

11 Maria Ntekoume

Title: Homogenization for the nonlinear Schrödinger equation with sprinkled nonlinearity.
Abstract: The nonlinear Schrödinger equation with sprinkled nonlinearity was introduced recently as a model for the propagation of waves with strong self-interaction at defects of the medium distributed according to a homogeneous Poisson point process. In this talk we will present a homogenization result for this model. If time permits, we will also discuss how the solutions fluctuate around the homogenized limit.

12 Martin Spitz

Title: The energy-critical stochastic Zakharov system
Abstract: The Zakharov system is a model in plasma physics describing rapid oscillations of the electric field in a conducting plasma. It consists of a Schrödinger and a wave equation with quadratic coupling. In this talk we consider the stochastic Zakharov system in the energy-critical dimension $d = 4$. We show local well-posedness up to the maximal existence time and prove that the solution exists globally as long as it does not cross the mass-energy threshold of the ground state. We also present a regularization by noise phenomenon, establishing that, for any initial data, the probability of global existence and scattering converges to one as the strength of the (non-conservative) noise goes to infinity. The talk is based on joint work with Sebastian Herr, Michael Röckner, and Deng Zhang.

13 Nicolas Camps

Title: Do Energy Cascade Mechanisms for NLS Persist on Generic Tori?
Abstract: When the nonlinear Schrödinger equation (NLS) is posed on the square torus, exact resonances can lead to energy transfers and growth of higher-order Sobolev norms. In this presentation, we investigate whether these mechanisms persist under perturbations of the domain, when the exact resonances are replaced by quasi-resonances. Under generic Diophantine conditions on the torus, we prove that energy cascades are absent over longer time scales compared to those expected on the

square torus. The proofs are based on normal form methods, and the analysis of the quasi-resonant interactions involves a frequency separation property. These are joint works with Joackim Bernier and Gigliola Staffilani.

14 Ruoyuan Liu

Title: Global dynamics and weak universality for the fractional hyperbolic Φ_3^4-model
Abstract: In this talk, I consider the 3-dimensional stochastic damped fractional nonlinear wave equation (with order $\alpha > 1$) with a cubic nonlinearity, also known as the fractional hyperbolic Φ_3^4-model. The construction of the fractional Φ_3^4-measure (Gibbs measure for the fractional hyperbolic Φ_3^4-model) exhibits a phase transition: when $\alpha > 9/8$, the Gibbs measure is equivalent with the base Gaussian measure; when $1 < \alpha \leq 9/8$, the Gibbs measure is mutually singular with the base Gaussian measure. I will first talk about global well-posedness of the fractional hyperbolic Φ_3^4-model and invariance of the fractional Φ_3^4-measure for the entire range $\alpha > 1$. Then, I will discuss weak universality for the fractional hyperbolic Φ_3^4-model. In the case $\alpha > 9/8$, we prove weak universality by using a first order expansion, invariance of Gibbs measures, and space-time analysis. In the case $1 < \alpha \leq 9/8$, we also obtain weak universality by overcoming the issue of singularity between the Gibbs measure and the base Gaussian measure.

15 Thierry Laurens

Title: Continuum Calogero–Moser models
Abstract: The focusing CCM model is a dispersive equation that describes a continuum limit of a particle gas interacting pairwise through an inverse square potential. Recently, Gérard and Lenzmann discovered solutions to this equation that exhibit turbulent behavior.
In this talk, we will discuss a scaling-critical well-posedness result for the focusing and defocusing CCM models on the line. In the focusing case, this requires solutions to have mass less than that of the soliton. This is joint work with Rowan Killip and Monica Visan.

16 Tristan Robert

Title: Regularization by noise for some modulated dispersive PDEs
Abstract: It is known since pioneering works by Veretennikov and Krylov - Röckner that for ODEs driven by a rough vector field, uniqueness of the solution can be recovered by adjunction of an additive noise in the equation. Improvement on the behavior of an ODE or PDE by adding a noise term is therefore referred to as a regularization by noise phenomenon, and is widely believed to hold for a large class of ODEs/PDEs and perturbative noises. In this talk, I will consider nonlinear dispersive PDEs where a deterministic noise is added as a distributional time coefficient in front of the dispersion. Despite the roughness of the noise term, we will see that any semilinear dispersive PDE with this noise term is well-posed at least in the same range of regularity as its noiseless counterpart, as soon as well-posedness relies on linear space-time estimates. Perhaps more surprisingly, provided that the noise is irregular enough, we will observe several regularization by noise phenomena: large data global well-posedness for focusing mass-critical equations, well-posedness at super-critical regularity for strongly non-resonant equations through improved multilinear estimates, and improvement on the Cauchy theory for Kadomtsev-Petviashvili equations through short-time multilinear estimates on longer time scales.

17 Yvonne Alama-Bronsard

Title: Numerical approximations to nonlinear dispersive equations in non-smooth regimes
Abstract: This talk deals with the numerical approximation to nonlinear dispersive equations, such as the prototypical nonlinear Schrödinger equation. We introduce novel integration techniques allowing for the construction of schemes which perform well both in smooth and non-smooth settings. We obtain symmetric low-regularity schemes with very good structure preserving properties over long times.

Higher order extensions will be presented, following new techniques based on decorated trees series inspired by singular stochastic PDEs via the theory of regularity structures. We will consider both the case of deterministic and randomized initial data.
If time permits, we will discuss very recent numerical advances for the approximation to completely integrable systems, such as the Benjamin-Ono equation.

18 Chenmin Sun

Title: Probabilistic well-posedness for the NLS on the two-sphere
Abstract: This talk addresses the probabilistic well-posedness for the nonlinear Schrödinger equation on the two-dimensional sphere. The initial data are distributed according to Gaussian measures with positive regularity in Sobolev spaces. The level of regularity goes beyond existing deterministic results, in a regime where the flow map cannot be extended uniformly continuously. In this talk, I will present the resolution ansatz as well as main steps of the proof. This is a joint work with Nicolas Burq, Nicolas Camps and Nikolay Tzvetkov.

19 (Participating) Dylan Samuelian

Title: Construction of blow-up solutions for nonlinear wave equation in low dimensions
Abstract: In this talk, I will consider the focusing, critical energy, nonlinear wave equation

$$\partial_{tt} u \Delta u - |u|^{p-1} u = 0, \quad t \geq 0, x \in \mathbb{R}, d \in \{3,4,5\}, p = \frac{(d+2)}{(d2)}$$

and explain how one can construct radial solutions which blow-up in finite time, while the energy of the solution remains finite (also known as a Type II blow-up). Such solutions are constructed as a perturbation of a blow-up profile $\lambda(t)^{\frac{(d2)}{2}} W(\lambda(t)|x|)$, which is a rescaling of a stationary solution $W(x)$ (called the ground state) with a polynomial blow-up rate $\lambda(t) = t^{-1\nu}, \nu \geq \nu_0$. The construction scheme, introduced first by Krieger-Schlag-Tataru, can be applied to other equations such as the 3D-critical NLS, Schrödinger maps or wave maps. It can also be adapted to nonlinearities which are not polynomials, showing its robustness.

20 (Participating) Matthias Ostermann

Title: Stable self-similar blowup for nonlinear wave equations beyond light cones
Abstract: We consider the corotational wave maps equation and equivariant Yang-Mills equation in all energy-supercritical dimensions. These are exemplary geometric wave equations and each of them exhibits an explicit self-similar blowup solution. As a matter of fact, both solutions continue to exist even past the singularity. In this talk, we present new theorems on the nonlinear asymptotic stability of these blowup solutions in spacetime regions that reach beyond the backward light cone and toward the future light cone of the singularity. The analysis is based on coordinate systems that are adapted to self-similarity and compatible with the wave evolution, and thereby allow to evolve the wave flow near the self-similar blowup in such spacetime regions.

21 (Participating) Xi Chen

Title: Introduction to the half wave Schrödinger equation
Abstract: The half wave Schrödinger equation is considered as a toy model motivated by the study of solutions to weakly dispersive and anisotropic equations, with one direction corresponding to the half wave operator, which is not dispersive. In this talk, we give an overview of the results on this equation, including the well-posedness results and the existence of modified wave operators in the defocusing case.

22 (Participating) Ambre Chabert

Title: Weakly turbulent solution to the Schrödinger equation on the two-dimensional torus with real potential decaying to zero at infinity.

Abstract: In this talk, I will construct a smooth solution to the linear Schrödinger equation on the 2D torus, perturbed by a smooth real potential vanishing as time tends to infinity, while the H^1 Sobolev norm of the solution grows logarithmically and hence it blows up in infinite time. I will present how I have been able to adapt the well-known techniques of CKSTT 2010, which relies on nonlinear interactions between Fourier modes to exhibit (finite) norm growth in the cubic nonlinear Schrödinger equation on the 2D torus, to a similar linear setting; thus, I will argue how the nonlinear ideas yield a new perspective on the linear problem.

Young researchers in deterministic and probabilistic dispersive equations

	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
9:30-10:30		JENDREJ	COLLOT	NTEKOUME	SANWAL
10:30-11:00	REGISTRATION	COFFEE BREAK	COFFEE BREAK	COFFEE BREAK	COFFEE BREAK
11:00-12:00	CAMPS	BRINGMANN	GLOGIC	LAURENS	SPITZ
12:00-13:00	COE	TOLOMEO	VALET	GASSOT	WALLAUCH-HAJDIN
13:00-14:30	LUNCH	LUNCH	X	LUNCH	X
14:30-15:30	ALAMA-BRONSARD	SUN	X	LIU	X
15:30-16:00	COFFEE BREAK	COFFEE BREAK	X	COFFEE BREAK	X
16:00-16:30	CHABERT	CHEN	X	ROBERT	X
16:30-17:00	OSTERMANN	SAMUELIAN	X		X
	X	X	X	X	X
17:30-19:00	X	POSTER+WINE	DINNER	X	X