# Perpetual Futures Pricing: Basic Formula

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In our paper (AHJ, 2023), we price various perpetual futures contracts under different assumptions. This page presents the pricing formula for the special case with constant interest rates. This basic formula is closely related to Covered Interest Parity used to price futures and forwards with finite maturities.

Define $x_t$ as the spot price of BTC in USD. The one-period riskless rates $r_a$ for USD and $r_b$ for BTC are assumed to be constant. For the most basic contract, entering a perpetual futures at time $t$ implies that the long side receives

$$(f_{t+1} - f_t) - \kappa (f_t - x_t)$$

at $t + 1$. The first part, $(f_{t+1} - f_t)$, is the same as for a classical futures contract. The second part, $\kappa (f_t - x_t)$, is the *funding* payment with $\kappa > 0$. This payment ensures that the futures price remains close to the spot price. Intuitively, if the futures price at time $t$ is above the spot price, the long side has to pay a positive amount $\kappa (f_t - x_t)$. This makes the long side relatively less attractive and contributes to lowering the futures price. Unlike for classical futures contracts, this contract does not have a maturity date, and unless it is closed out (by an offsetting position), it goes on in perpetuity. This specification has the advantages that no rollover of maturing contracts is required and that liquidity is concentrated in a single contract.

As shown in AHJ, ruling out arbitrage and under a no-bubble condition, the **price of the perpetual futures** satisfies

$$f_t = \frac{\kappa (1 + r_b)}{\kappa (1 + r_b) - (r_a - r_b)} x_t. \tag{1}$$

In contrast, the classical one period futures/forward contract through which the long receives $\left(x_{t+1} - f_t^{(1)}\right)$ has a no-arbitrage price

$$f_t^{(1)} = \left(\frac{1 + r_a}{1 + r_b}\right) x_t,$$

the well-known Covered Interest Parity equation. In both cases, the futures price exceeds the spot price to the extent that $r_a$ exceeds $r_b$, that is the numeraire interest rate exceeds the interest rate of the underlying asset. The typical period length for perpetual futures settlement payments is 8 hours. As implied by equation (1), with $\kappa$ having a value close to 1 and effective interest rates for an 8-hour period, the no-arbitrage price of a perpetual contract should always be very close to the spot price.

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