Roundtable on Default Risk Correlation Models

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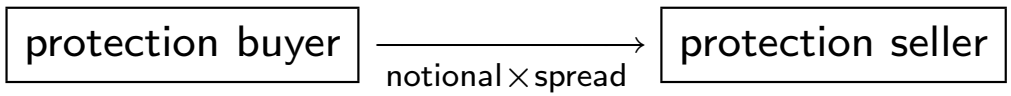
- CDS/CDX Market
- CDO Market
- Some empirical evidence
- Final Thoughts

Credit markets characterized by rapid financial innovation

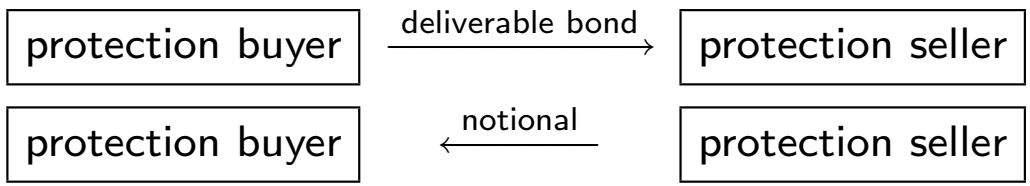
- Innovation in contracts,
  - from traditional *funded* securities: corporate bonds
  - to new *unfunded* derivatives: credit default swaps (CDS)
- And increased liquidity,
- Allow investors to express views on:
  - Single-names CDS
  - Baskets of names (CDX.IG, CDX.HV, iTraxx)
  - Correlation (Synthetic liquid CDO, Bespoke CDO, $CDO^2 \ldots$)
  - Emerging Market Countries (EMCDS)
  - Basket of Countries (EMCDX)
  - Asset Backed Securities such as credit card receivables or Home equity loans (ABS-CDS)
  - Baskets of Asset Backed Securities (ABX)
  - Correlation (TABX)
  - Senior secured Loans (LCDS)
  - Basket of Loans (LCDX)

CDS Contract Structure

- A CDS is an insurance contract against a credit event of Counterparty:
  - Prior to credit event:



  - Upon arrival of credit event:



- Definition of credit event:

  Bankruptcy
  Failure to pay
  Obligation acceleration or default
  Repudiation/moratorium
  Restructuring (Full R, Mod R, ModMod R, No R)

Arbitrage Relation

- Buy XYZ bond + Buy XYZ protection $\sim$ Earn risk-free rate
- Buy risk-free bond + Sell XYZ protection $\sim$ Earn XYZ bond yield

$$\boxed{\text{CDS spread} \approx Y_{XYZ} - R_f}$$

$\Rightarrow$ CDS allows pure unfunded play on credit risk.

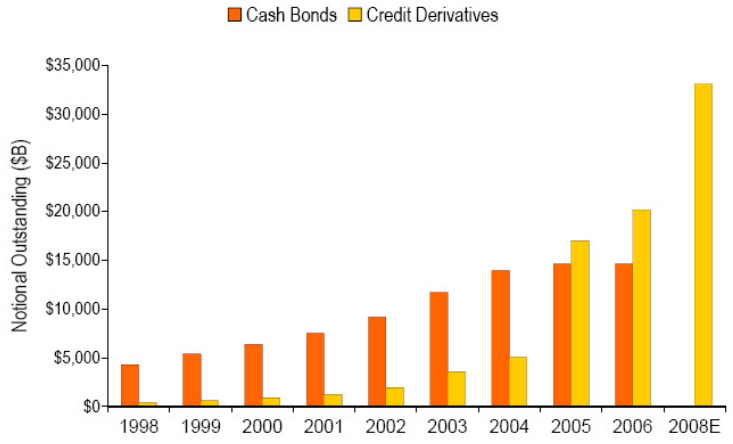
- Empirical evidence on *Basis* = CDS spread $-$ $(Y_{XYZ} - R_f)$.

	Basis wrt Tsy (bp)		Basis wrt Swap (bp)		implied R_f / Tsy	
	Mean	S.E. (of mean)	Mean	S.E.	Mean	S.E.
Aaa/Aa	-51.30	1.97	9.55	1.31	0.834	0.0250
A	-64.33	1.82	5.83	1.59	0.927	0.0229
Baa	-84.93	3.63	2.21	2.79	0.967	0.0364
All Categories	-62.87	1.38	6.51	1.06	0.904	0.0160

source: Hull, Pedrescu, White (2006)

CDS Market Statistics

Exhibit 1.1: The notional amount of credit derivatives globally is larger than the global amount of debt outstanding



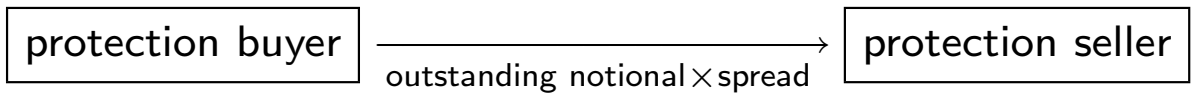
Sources: British Bankers' Association Credit Derivatives Report 2006, Bank for International Settlements and ISDA.
Note: Cash bonds through June 2006.

Exhibit 7.1: Participants in the credit derivatives market. Some favor one direction over the other.

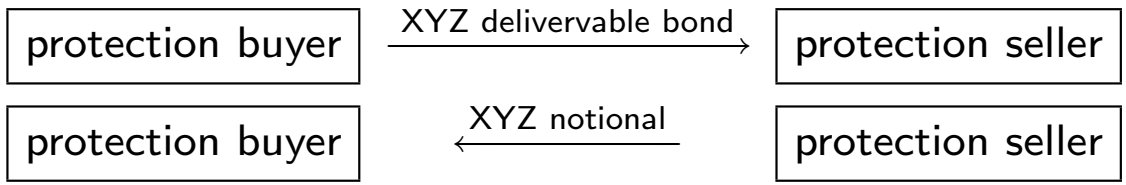
Source: *British Bankers' Association Credit Derivatives Report 2006.*

The CDX index

- The CDX index is an insurance contract against credit events of a portfolio of counterparties (e.g., 125 names in CDX.IG):
  - Prior to credit event:



  - Upon arrival of credit event of XYZ:

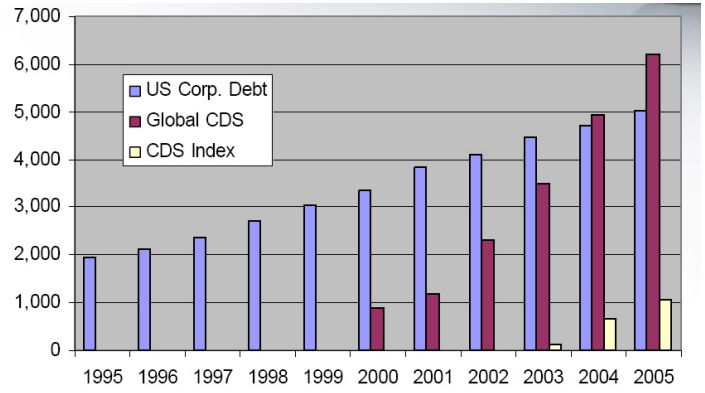


  - Following credit event outstanding notional is reduced by notional of XYZ in portfolio (i.e., $\frac{1}{125}$ in CDX.IG).
  - Contract expires at maturity or when notional exhausted.

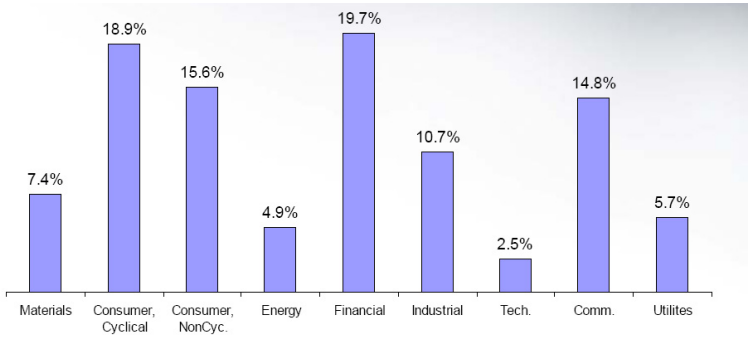
- N.B.: CDX contract $\neq$ equally weighted portfolio of single name CDS contracts
  CDX spread $\neq$ average of single name CDS spreads

CDX Market Statistics

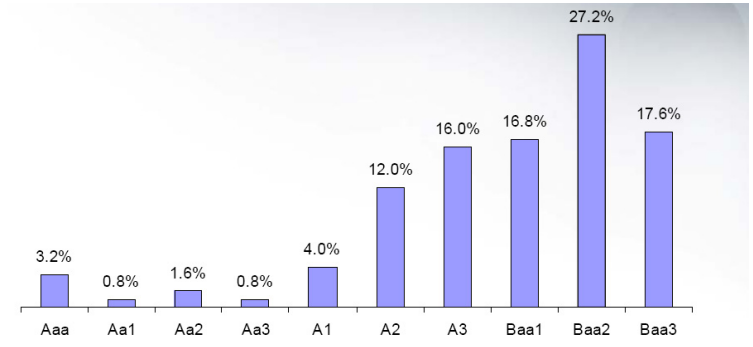
Growth Rate (notional)



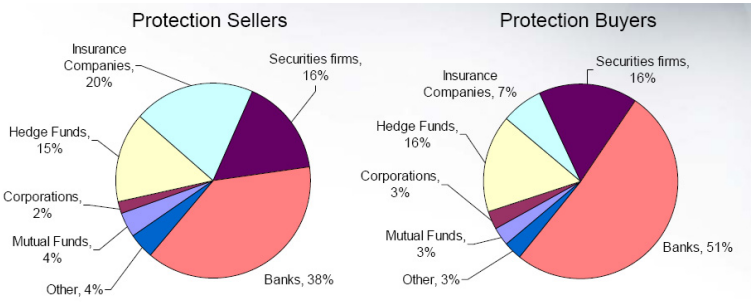
Industry Composition of CDX.IG



CDX.IG Moody's Ratings



End Users



source: BBA & White (2006)

Synthetic CDO Tranches

- Selling protection on CDO tranche with attachment points $[L, U]$ (i.e., notional $= U - L$) written on underlying basket of 125 single names (CDX):
  - Prior to a credit event:

$$\boxed{\text{protection buyer}} \xrightarrow[\text{outstanding notional} \times \text{spread}]{} \boxed{\text{protection seller}}$$

  - Upon arrival of credit event ($LGD =$ notional $-$ deliverable bond price), if cumulative loss exceeds lower attachment point (i.e., $\mathcal{L}_t = \sum_{i=1}^{125} LGD_i \mathbf{1}_{\{\tau_i \leq t\}} > L$) then

$$\boxed{\text{protection buyer}} \xleftarrow{\min(LGD, \text{outstanding notional})} \boxed{\text{protection seller}}$$

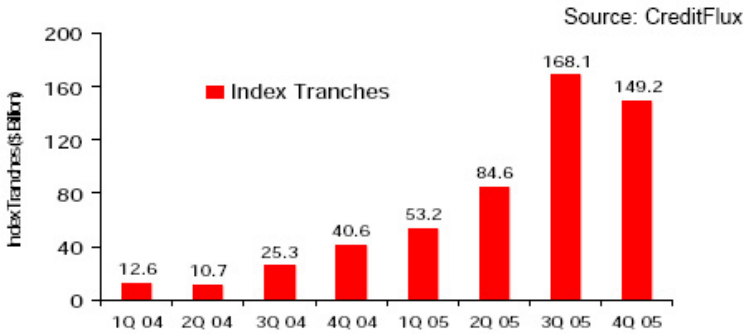
  - Following credit event outstanding tranche notional is reduced by LGD (up to exhaustion of outstanding notional).
  - Also, super senior tranche notional is reduced by recovery (to satisfy 'adding up constraint').
  - Contract expires at maturity or when tranche notional is exhausted.
- Tranche payoff is call spread on cumulative loss: $\max(\mathcal{L}_t - L, 0) - \max(\mathcal{L}_t - U, 0)$.

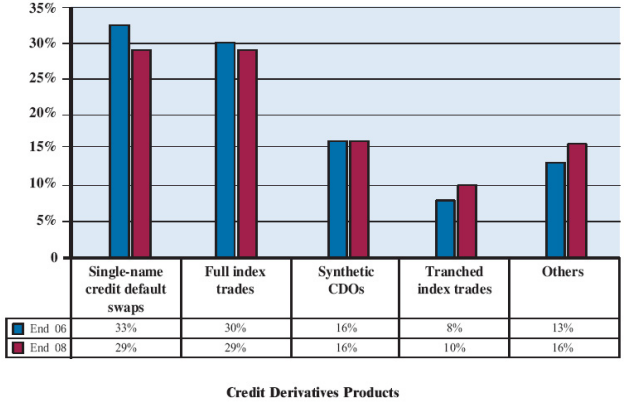
$\Rightarrow$ Tranche valuation depends on entire distribution of cumulative portfolio losses and crucially on default event correlation model.

Market Size

- Liquid tranche market is growing steadily

Type	2000	2002	2004	2006
Basket products	6.0%	6.0%	4.0%	1.8%
Credit linked notes	10.0%	8.0%	6.0%	3.1%
Credit spread options	5.0%	5.0%	2.0%	1.3%
Equity linked credit products	n/a	n/a	1.0%	0.4%
Full index trades	n/a	n/a	9.0%	30.1%
Single-name credit default swaps	38.0%	45.0%	51.0%	32.9%
Swaptions	n/a	n/a	1.0%	0.8%
Synthetic CDOs – full capital	n/a	n/a	6.0%	3.7%
Synthetic CDOs – partial capital	n/a	n/a	10.0%	12.6%
Tranched index trades	n/a	n/a	2.0%	7.6%
Others	41.0%	36.0%	8.0%	5.7%





- Bespoke portfolio tranche market is much larger (ten times?) than synthetic tranche market:
  - Investors sell or buy protection on a portfolio of specific names for speculative or hedging motives.
  - Dealers take the other side and turn to the synthetic tranche market to hedge their resulting net exposure (keep some basis risk).
  - Hedge funds and other dealers participate in synthetic tranche market to redistribute risks.

Market Model: Implied Gaussian Copula Correlation

- Market standard for quoting CDO tranche prices is the *implied correlation* of the Gaussian Copula framework.

- Intuition builds on structural model of default (CDO model due to Vasicek 1987 who combines Merton (1974) with CAPM idea):
  - Each name in basket characterized by an 'asset value' driven by two factors: a common market factor and an idiosyncratic factor ($V_i = \sqrt{\rho_i}\, M + \sqrt{1-\rho_i}\, \epsilon_i$ with M, ϵ_i independent centered Gaussian).
  - Pairwise 'asset correlation' is the product of the individual asset betas ($\sqrt{\rho_i \rho_j}$).
  - Default occurs when asset value falls below a constant barrier (DefProb $= P(V_i \leq B_i)$).

- Market convention for quoting tranche values in terms of *implied correlation* assumes:
  - The individual beta is identical across all names in the basket.
  - The default boundary is identical and calibrated to CDX level.
  - All firms have identical LGD of 60%.

$\Rightarrow$ With these heroic assumptions, a single number, the *implied correlation* ($= \rho$), allows to match a given tranche's model price with the market price (for a given CDX level).

The implied correlation smile

- Market Quotes on Aug. 4, 2004 (CDX index spread 63.25 bp)

Tranche	0-3%	3-7%	7-10%	10-15%	15-30%
CDX.IG	41.38%	3.49%	1.355%	0.46%	0.14%

- The market displays an *implied correlation smile*:

Imp Corr	21.7%	4.1%	17.8%	18.5%	29.8%

$\Rightarrow$ The smile shows that the Gaussian copula model is mis-specified ($\sim$ option skew).

- Market quotes on June 1st 2005 IG4-5Y (CDX index spread of 42 bp):

Tranche	0-3%	3-7%	7-10%	10-15%	15-30%
CDX.IG	30.5%	0.66%	.095%	.075%	0.04%
Imp Corr	9.08%	5.8%	10.02%	16.77%	27.62%

- Market quotes on June 4, 2008 IG9-5Y (CDX index ref 118 bp):

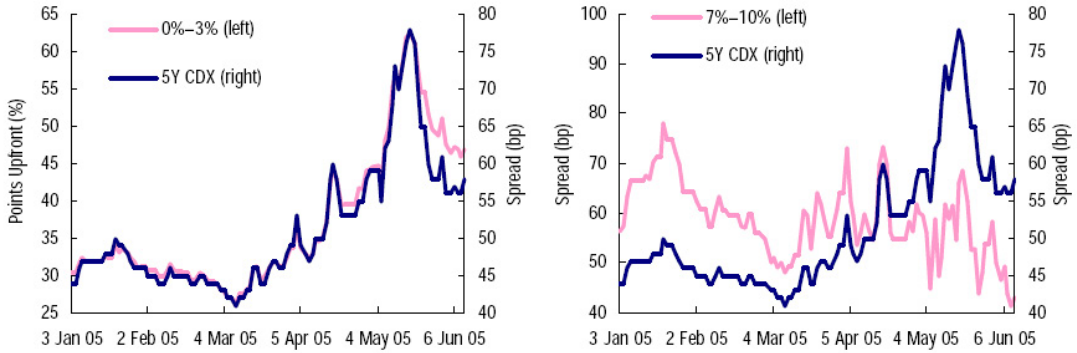
Tranche	0-3%	3-7%	7-10%	10-15%	15-30%	30-100%
CDX.IG	51.5%	4.35%	2.32%	1.3%	0.70%	0.41%
Imp Corr	40%	88.23%	4.31%	13.47%	32.06%	88.35%

Correlation 'trading'

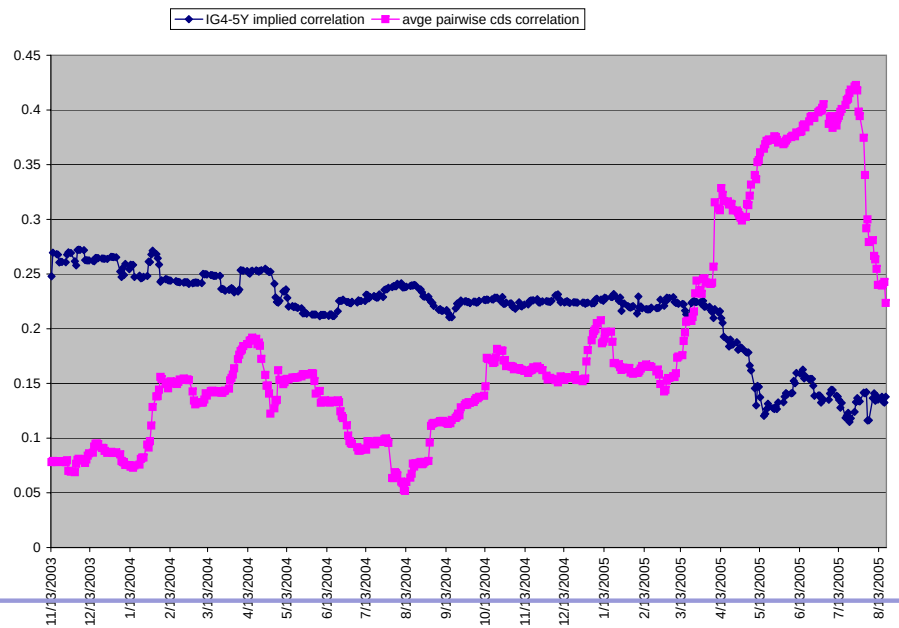
- Selling protection on the equity Tranche (delta-hedged) $\sim$ long correlation:
  - Selling protection on equity is equivalent to being long a put on aggregate losses with strike equal to 3%. The value is increasing in the volatility of total losses which increases with default correlation.
  - The equity tranche is exposed to idiosyncratic Jump-to-default risk since it gets hit at the first default.
- Selling protection on the senior tranches $\sim$ short correlation:
  - Selling protection on super senior tranche is short a call option on aggregate portfolio losses struck at 30%. Its value is decreasing in loss volatility and hence decreasing in correlation.
  - The Super senior tranche is exposed to systematic (cataclysmic?) risk: What is the probability that $>$ 30% of investment grade default within a year?
- At least two reasons for the rapid development of CDS/CDX/CDO markets:
  - Credit spread puzzle
  - Rating 'arbitrage'

May 2005 'repricing' of correlation risk

- Events in May 2005 (widening of GM and Ford) had dramatic impact on tranche prices:

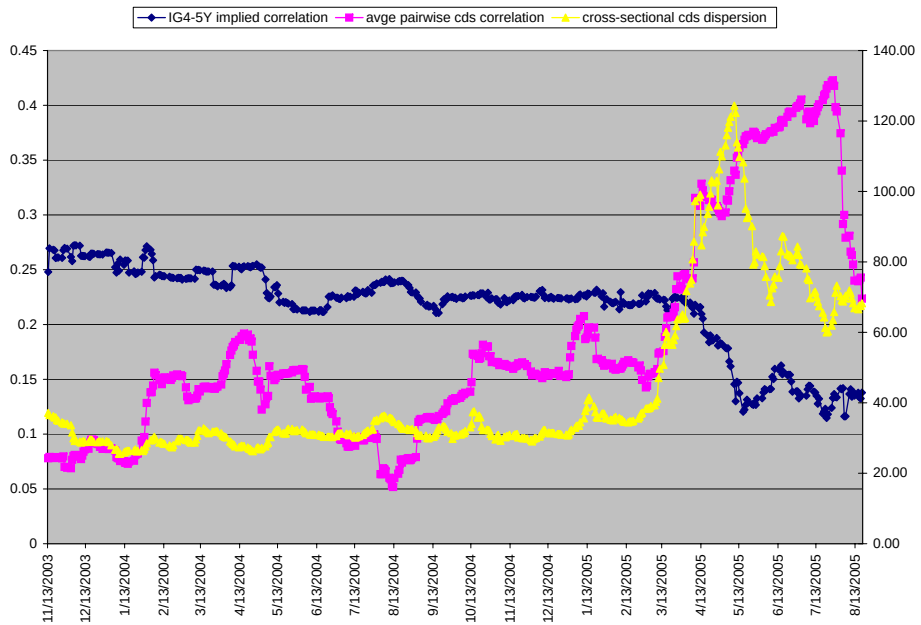


- As a result, 'repricing' in correlation markets (equity implied correlation dropped from 20% to 10%). Yet, measures of actual (e.g., spread) correlation increased:



Implied correlation: What does it measure?

- May 2005 'repricing' in correlation markets: impact of cross-sectional dispersion?

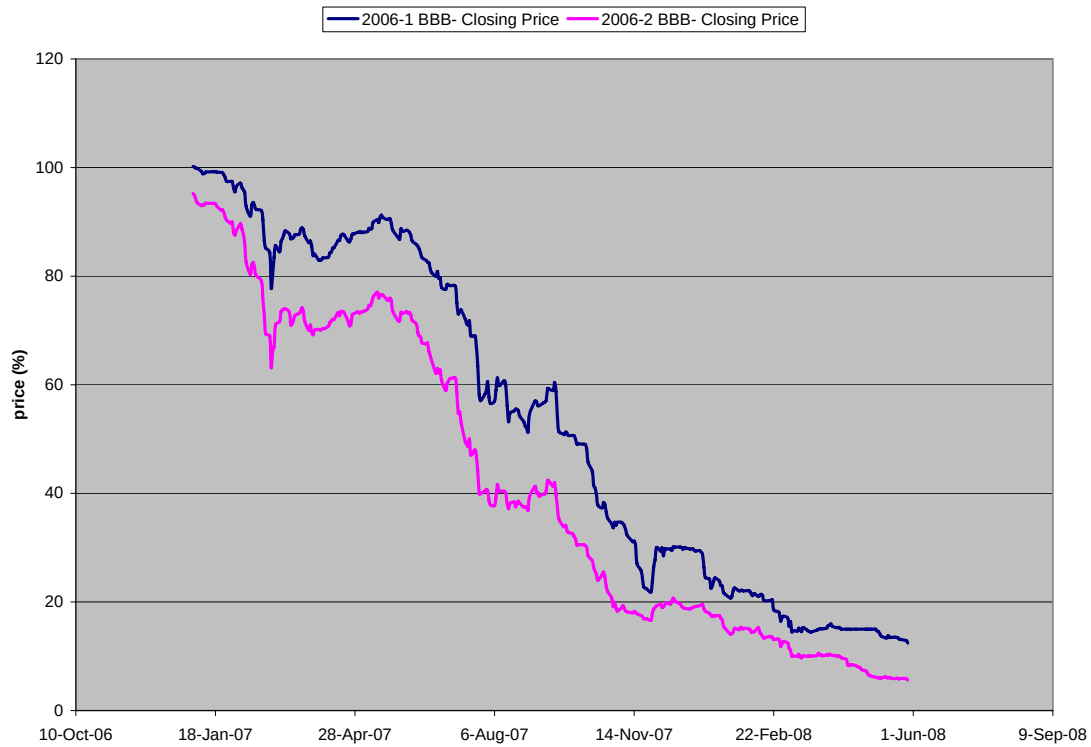


- Trading equity implied correlation $\approx$ trading jump to default risk.
  - selling protection on IG4 equity in May 2005 essentially sells protection on first to default basket of autos.

- Trading senior tranches implied correlation $\approx$ market crash/great depression risk.
  - What is the probability that $>$ 30% of investment grade default in any given year?

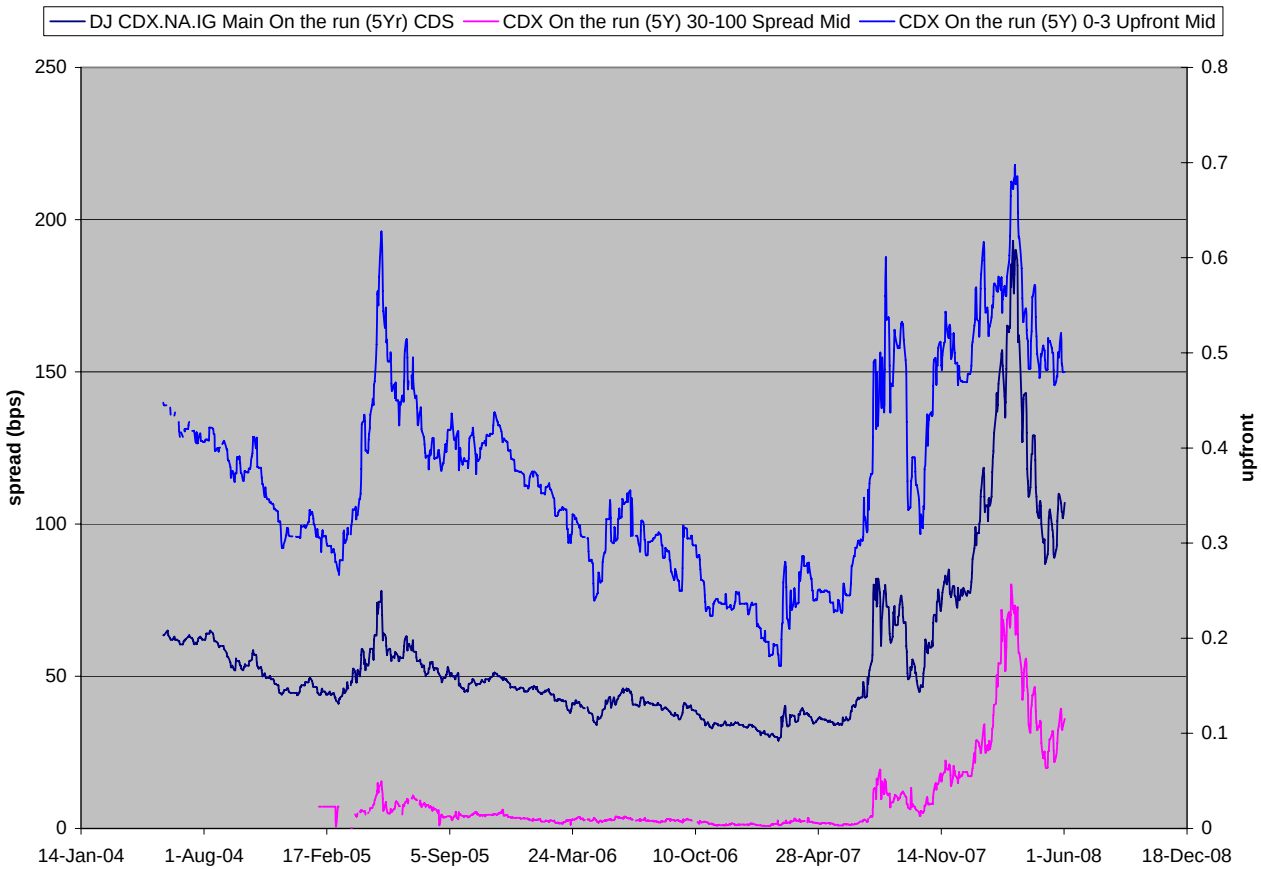
The impact of Subprime on Correlation Markets

- Subprime hits in February 2007, then accelerates in June-July 2007.



The impact of Subprime on Correlation Markets

- The impact on tranche prices was dramatic



- Implied correlation on equity tranche hit $> 40\%$
- Correlation on Super-Senior tranches $> 1(!)$ with standard recovery assumption

$\Rightarrow$ Cheap solution: set recovery to zero! But is that realistic?

Implications for modeling correlation

- Need a better modeling framework (beyond Gaussian Copula):
  - Implied Gaussian copula correlation is not a good indicator of correlation
  - There is no corresponding measure of 'realized correlation' ($\neq$ implied option volatility)
  - Predicted hedges don't work well during volatile periods.
  - The model is inherently static (one-period).

- Several alternative models have been proposed:
  - Bottom-up (can be fit to individual constituents characteristics)
    - Reduced-form models (Duffie-Garleanu, Mortensen)
    - Extensions of standard Copula framework to multiple factors, non-Gaussian copula, random recovery, Implied Copula... (Andersen-Sidenius, Hull-White).
  - Top down (model aggregate losses without reference to constituents' characteristics)
    - Reduced-form approach (Longstaff-Rajan, Schönbucher)

- What one would like:
  - Predictive Model (i.e., calibrated to observables that delivers consistent pricing of **all** tranches).
  - Constituents' Spread dynamics should be an input (level, cross-sectional dispersion, volatility).
  - Models should deliver hedging/risk-measures of Jump-to-default risk **and** spread/marking-to-market risk.
    (for Super-Senior, most risk comes from spread risk).