Discussion of "An empirical analysis of the pricing of collateralized Debt obligation" by Francis Longstaff and Arvind Rajan

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- Summary
- The CDS/CDX Market
- The CDO Market
- New modeling approach
- Empirical implementation
- Conclusion

Summary of the paper

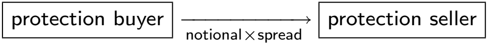
- This paper studies an interesting new data set on a new market: Synthetic CDO tranches
- It develops an elegant reduced form model in the 'spirit' of Duffie & Garleanu
- It empirically fits the model to the data by minimizing sum of squared errors and finds:
  - Three 'factors' are needed to fit tranche spreads on five tranches.
  - These are three stochastic intensity processes that govern the default arrival of respectively:
    - Single firm default (1 firm defaults on average every 1.2 years)
    - Joint industry wide defaults (15 firms default jointly on average every 42.5 years)
    - Economy wide defaults (88 firms default jointly on average every 763 years)
  - The model fit is very good. The RMSE is around 3 to 5 bps.
- Paper concludes that *"Pricing in these markets is highly efficient. This is true even during the credit crisis of May 2005 which resulted in major losses for a number of major credit-oriented hedge funds."*

Rapid evolution of credit markets

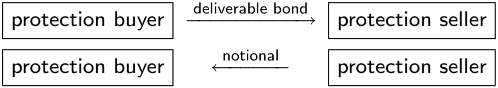
- Innovation in contracts,
  - from traditional *funded* securities: corporate bonds
  - to new *unfunded* derivatives: credit default swaps (CDS)
- And increased liquidity,
- Allow investors to express views on:
  - Single-names CDS
  - Baskets of names (CDX.IG, CDX.HV, iTraxx)
  - Correlation (Synthetic liquid CDO, Bespoke CDO, CDO^2...)
  - Emerging Market Countries (EMCDS)
  - Basket of Countries (EMCDX)

CDS Contract Structure

- A CDS is an insurance contract against a credit event of counterparty:
  - Prior to credit event:



  - Upon arrival of credit event:



- Definition of credit event:

  Bankruptcy
  Failure to pay
  Obligation acceleration or default
  Repudiation/moratorium
  Restructuring (Full R, Mod R, ModMod R, No R)

Arbitrage Relation

- Buy XYZ bond + Buy XYZ protection $\sim$ Earn risk-free rate
- Buy risk-free bond + Sell XYZ protection $\sim$ Earn XYZ bond yield

$$\boxed{\text{CDS spread} \approx \text{Y}_{XYZ} - \text{R}_f}$$

$\Rightarrow$ CDS allows pure unfunded play on credit risk.

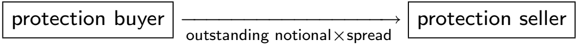
- Empirical evidence on *Basis* = CDS spread $-$ ($\text{Y}_{XYZ} - \text{R}_f$).

	Basis wrt Tsy (bp)		Basis wrt Swap (bp)		implied R_f/ Tsy	
	Mean	S.E. (of mean)	Mean	S.E.	Mean	S.E.
Aaa/Aa	-51.30	1.97	9.55	1.31	0.834	0.0250
A	-64.33	1.82	5.83	1.59	0.927	0.0229
Baa	-84.93	3.63	2.21	2.79	0.967	0.0364
All Categories	-62.87	1.38	6.51	1.06	0.904	0.0160

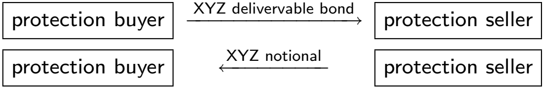
source: Hull, Pedrescu, White (2006)

The CDX index

- The CDX index is an insurance contract against credit events of a portfolio of counterparties (e.g., 125 names in CDX.IG):
  - Prior to credit event:



  - Upon arrival of credit event of XYZ:

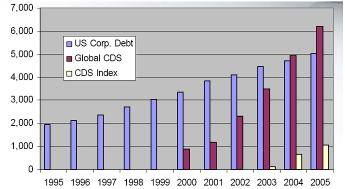


  - Following credit event outstanding notional is reduced by notional of XYZ in portfolio (i.e., $\frac{1}{125}$ in CDX.IG).
  - Contract expires at maturity or when notional exhausted.

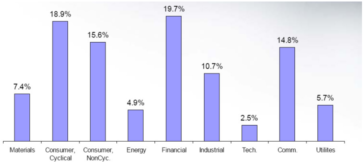
- N.B.: CDX contract $\neq$ equally weighted portfolio of single name CDS contracts
  CDX spread $\neq$ average of single name CDS spreads

Market Overview

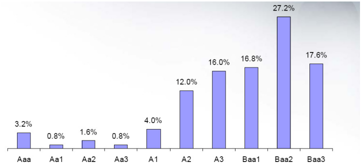
Growth Rate (notional)



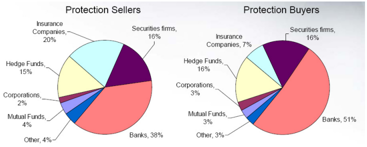
Industry Composition of CDX.IG



CDX.IG Moody's Ratings



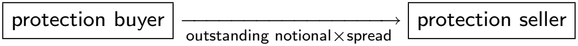
End Users



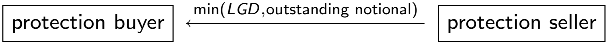
source: BBA & White (2006)

Synthetic CDO Tranches

- Selling protection on CDO tranche with attachment points $[L, U]$ (i.e., notional $= U - L$) written on underlying basket of 125 single names (CDX):
  - Prior to a credit event:



  - Upon arrival of credit event ($LGD =$ notional $-$ deliverable bond price), if cumulative loss exceeds lower attachment point (i.e., $\mathcal{L}_t = \sum_{i=1}^{125} LGD_i \mathbf{1}_{\{\tau_i \leq t\}} > L$) then

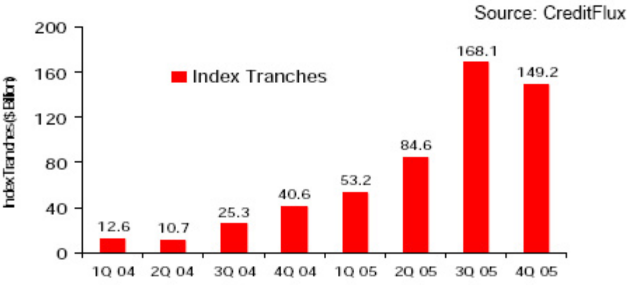


  - Following credit event outstanding tranche notional is reduced by LGD (up to exhaustion of outstanding notional).
  - Contract expires at maturity or when tranche notional is exhausted.
- Tranche payoff is call spread on cumulative loss: $\max(\mathcal{L}_t - L, 0) - \max(\mathcal{L}_t - U, 0)$.

$\Rightarrow$ Tranche valuation depends on entire distribution of cumulative portfolio losses and crucially on default event correlation model.

Market Size

- Liquid tranche market is growing steadily



- Bespoke portfolio credit swap market is roughly ten times the size of the index tranche market.

Market Model: Implied Gaussian Copula Correlation

- Market standard for quoting CDO tranche prices is the *implied correlation* of the Gaussian Copula framework.
- Intuition builds on structural model of default (CDO model due to Vasicek 1987):
  - Each name in basket characterized by an 'asset value' driven by two factors: a common market factor and an idiosyncratic factor ($V_i = \sqrt{\rho_i}\, M + \sqrt{1-\rho_i}\, \epsilon_i$ with M, ϵ_i independent centered Gaussian).
  - Pairwise 'asset correlation' is the product of the individual asset betas ($\sqrt{\rho_i \rho_j}$).
  - Default occurs when asset value falls below a constant barrier (DefProb $= P(V_i \leq B_i)$).
- Market convention for quoting tranche values in terms of *implied correlation* assumes:
  - The individual beta is identical across all names in the basket.
  - The default boundary is identical and calibrated to average CDS level (or index level)
  - All firms have identical LGD of 60%.

⇒ With these heroic assumptions, a single number, the *implied correlation* ($= \rho$), allows to match a given tranche's model price with the market price (for a given index CDS level).

The implied correlation smile

- Market Quotes on Aug. 4, 2004 (CDX index spread 63.25 bp)

Tranche	0-3%	3-7%	7-10%	10-15%	15-30%
CDX.IG	41.38%	3.49%	1.355%	0.46%	0.14%

- The market displays an *implied correlation smile*:

Tranche	0-3%	3-7%	7-10%	10-15%	15-30%
CDX.IG	21.7%	4.1%	17.8%	18.5%	29.8%

⇒ The smile shows that the Gaussian copula model is mis-specified (analogous to the implied option smile).

- Market quotes on June 1st IG4-5Y (CDX index spread of 42 bp):

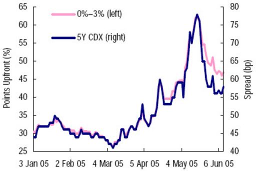
Tranche	0-3%	3-7%	7-10%	10-15%	15-30%
CDX.IG	30.5%	0.66%	.095%	.075%	0.04%

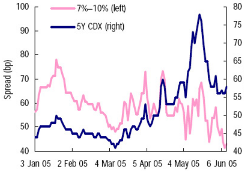
- The current *implied correlation smile*:

Tranche	0-3%	3-7%	7-10%	10-15%	15-30%
CDX.IG	9.08%	5.8%	10.02%	16.77%	27.62%

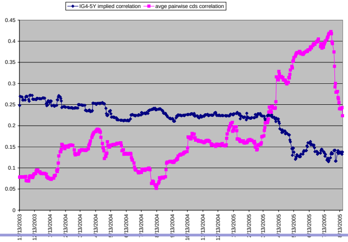
Failure of Copula Model?

- Events in May 2005 (widening of GM and Ford) had dramatic impact on tranche prices: Equity ([0,3%]) and index ([0,100%]) widened, while Mezz ([3%,7%]) tightened!



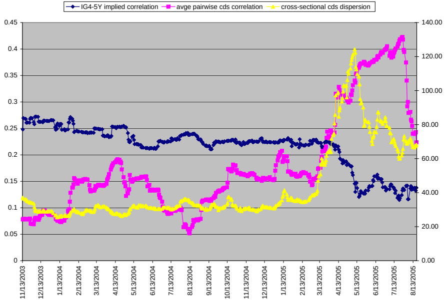


- As a result, 'repricing' in correlation markets (equity implied correlation dropped from 20% to 10%). Yet over the same period measures of actual correlation increased:



Looking for better model?

- May 2005 'repricing' in correlation markets: impact of cross-sectional dispersion?



- Trading equity implied correlation ≈ trading jump to default risk.
  - selling protection on IG4 equity in May 2005 essentially sells protection on first to default basket of autos.

- Trading senior tranches implied correlation ≈ market crash/great depression risk.
  - What is the probability that $> 30\%$ of investment grade default in any given year?

Reduced-form model with heterogeneous firms

- Reduced-form approach (Duffie Garleanu (2001), Mortensen (2006))
  - Assume an intensity process for each underlying name:

$$\lambda_i(t) = \rho_i M(t) + \beta_i I(t) + \epsilon_i(t)$$

    where
    - $M(t)$ is market wide default intensity.
    - $I(t)$ is industry default component.
    - $\epsilon_i(t)$ is firm specific component.
  - Defaults are conditionally independent (doubly stochastic), but there is correlation in default arrival times through M and I.
  - Advantage:
    - conditionally independent defaults (not assumed to arrive jointly).
    - individual hedge ratios can be computed (i.e., impact of widening of GM or Ford).
    - Bespoke can be priced consistently
  - Disadvantage:
    - Cumbersome to implement (lots of parameters and state variables).
    - Difficult to calibrate.

Reduced-form model with homogeneous firms

- This paper proposes simple model of aggregate portfolio losses (assuming homogeneous firms):

$$\mathcal{L}_t = 1 - \exp\left(-\gamma_1 N_{1t} - \gamma_2 N_{2t} - \gamma_3 N_{3t}\right)$$

  - N_{1t} counts individual firm defaults ($\gamma_1 = 1/125$)
  - N_{2t} counts number of industry wide *simultaneous* defaults.
  - N_{3t} counts number of economy wide *simultaneous* defaults.
- Each driven by stochastic intensity process:

$$d\lambda_i(t) = \sigma_i \sqrt{\lambda_i(t)} dZ_{it}$$

- Advantage:
  - Simplicity of implementation/computation
- Disadvantage
  - Assumes joint defaults (to create correlation)
  - Difficult to compute individual name hedge ratios ($\neq$ analogy to S&P500 index option).
  - Difficult to apply to bespoke portfolios.
- Technical (minor) issues:
  - Absorption at zero of intensity
  - Intensity unchanged upon default arrival?

Approach

- choose the three intensity processes λ_{it} every day to minimize the cross-sectional fitting error of running spreads on five liquid tranches ($[0-3], [3-7], [7-10], [10-15], [15-30]$) as well as the index.

- In addition pick the three volatility parameters σ_i and three 'jump upon default' parameters γ_i.

- Allow all parameters to change for every CDX series (i.e., every 6 months). However, note that
  - Difference between IG3-IG4 series is 3 names,
  - IG4-IG5 is 9 names,
  - IG5-IG6 is 4 names

Question/Comments

- Why work with spreads?
  - Need to transform upfront payment on the equity in running spread? (model dependent)
  - Magnitude differences are huge: equity spread $\approx$ 2000bps whereas senior tranche $\approx$ 4 bps.
  - $\Rightarrow$ Minimization of sum of squared errors puts too much weight on equity and mezz fitting.
  - $\Rightarrow$ RMSE of 5 bps is very good for the equity tranche, but how meaningful for senior tranches?
  - $\Rightarrow$ How about fitting implied correlations $\sim$ using implied vols for out of the money options.
- Time series implications of the model?
  - Since three state variables are fitted every day, clearly can fit three prices perfectly $\Rightarrow$ only 2 out of sample points.
  - Parameters of state vector reset every series (despite the fact that at most a few names change at roll).
  - $\Rightarrow$ Necessity to bring in time series information.
  - How likely is it to generate these time series through simulation of assumed continuous time process?

Question/Comments

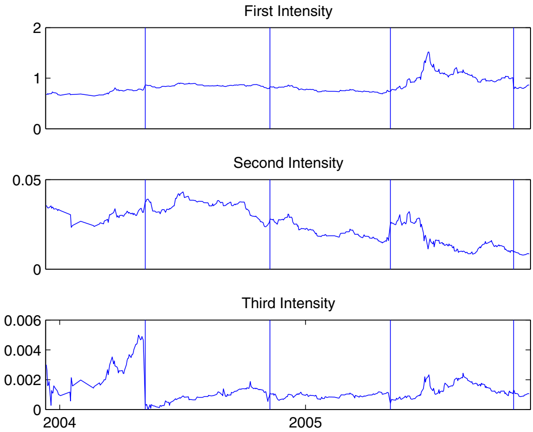


Fig. 3. Intensity Processes. This figure graphs the estimated intensity processes. The vertical division lines denote the roll from one CDX index to the next.

Is the CDO tranche market efficient?

- I don't know! But it seems an ideal candidate *not* to be:
  - It is a new market (cf. early days of option market or futures market).
  - It is not a transparent market (OTC - still some disagreement on settlement procedures).
  - It is a complicated product (payoff depends on higher order moments of portfolio losses).
  - There is very little data to work with (default data is scarce, but needed to estimate entire joint default distribution).
  - There is no market consensus about the model (post-May consensus is to retain Gaussian Copula model solely as quoting tool).
  - It is affected by "technicals," i.e., pipeline of issuances in bespoke CDO and cash CDO markets that trigger hedging demand by broker/dealers.
- What would be a convincing test of market (in)efficiency?
  - Seems difficult to uncover pure arbitrage (incomplete market/pricing by replication difficult).
  - Need to look at pricing kernel: Are there high sharpe ratio strategies/ good deals?
    - Pre-May 2005 selling protection on equity tranche is negative IR strategy assuming historical default and spread history.

Conclusion

- Very interesting new data on new market.
- Very elegant simple modeling approach.
- More to be done on the empirical front:
  - Avoid equally weighting spreads RMSE.
  - Take advantage of time series dimension of model.
  - What is risk-return tradeoff in tranche market?
  - What are hedging possibilities offered by model?