

“Do (should) Credit Index market conventions have an impact on prices?”

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Moody's-KMV Maarc - May 2007

- The CDS/CDX Market
- The CDX-CDS Basis
- Default Swaptions
- Bonus track: Venezuela case study

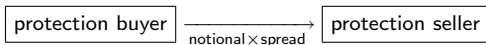
Rapid evolution of credit markets

- ▶ Innovation in contracts,
 - ▶ from traditional *funded* securities: corporate bonds
 - ▶ to new *unfunded* derivatives: credit default swaps (CDS)
- ▶ And increased liquidity,
- ▶ Allow investors to express views on:
 - ▶ Single-names CDS
 - ▶ Baskets of names (CDX.IG, CDX.HV, iTraxx)
 - ▶ Spread volatility (Spread options)
 - ▶ Correlation (Synthetic liquid CDO, Bespoke CDO, CDO²...)
 - ▶ Emerging Market Countries (EMCDS)
 - ▶ Basket of Countries (EMCDX)

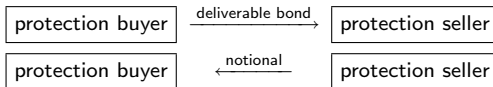
CDS Contract Structure

- ▶ A CDS is an insurance contract against a credit event of counterparty:

- ▶ Prior to credit event:



- ▶ Upon arrival of credit event:



- ▶ Definition of credit event:

Bankruptcy

Failure to pay

Obligation acceleration or default

Repudiation/moratorium

Restructuring (Full R, Mod R, ModMod R, No R)

Arbitrage Relation

- ▶ Buy XYZ bond + Buy XYZ protection \sim Earn risk-free rate
- ▶ Buy risk-free bond + Sell XYZ protection \sim Earn XYZ bond yield

$$\text{CDS spread} \approx Y_{XYZ} - R_f$$

\Rightarrow CDS allows pure unfunded play on credit risk.

- ▶ Empirical evidence on $\text{Basis} = \text{CDS spread} - (Y_{XYZ} - R_f)$.

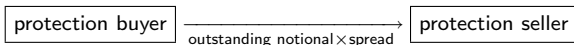
	Basis wrt Tsy (bp)		Basis wrt Swap (bp)		implied R_f / Tsy	
	Mean	S.E. (of mean)	Mean	S.E.	Mean	S.E.
Aaa/Aa	-51.30	1.97	9.55	1.31	0.834	0.0250
A	-64.33	1.82	5.83	1.59	0.927	0.0229
Baa	-84.93	3.63	2.21	2.79	0.967	0.0364
All Categories	-62.87	1.38	6.51	1.06	0.904	0.0160

source: Hull, Pedrescu, White (2006)

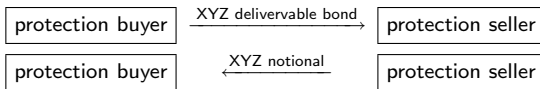
The CDX index

- ▶ The CDX index is an insurance contract against credit events of a portfolio of counterparties (e.g., 125 names in CDX.IG):

- ▶ Prior to credit event:



- ▶ Upon arrival of credit event of XYZ:



- ▶ Following credit event outstanding notional is reduced by notional of XYZ in portfolio (i.e., $\frac{1}{125}$ in CDX.IG).
- ▶ Contract expires at maturity or when notional exhausted.
- ▶ Question: What is the (no-arbitrage) relation between the CDX spread and the underlying single-name CDS spreads?

Portfolio of CDS (clearly \neq CDX)

- ▶ Single name CDS balances expected coupons and expected loss:

$$\mathbb{E}^Q \left[\int_0^T e^{-\int_0^t r(u)du} \text{CS}_i \mathbf{1}_{\{\tau_i > t\}} dt \right] = \mathbb{E}^Q \left[\int_0^T e^{-\int_0^t r(u)du} \ell_i d\mathbf{1}_{\{\tau_i \leq t\}} \right] \quad (1)$$

- ▶ Single-name CDS is weighted average forward losses:

$$\text{CS}_i = \int_0^T \omega_i(t) \lambda_i^f(t) \ell_i dt$$

where:

- ▶ the risk-free zero-coupon bonds: $D(0, t) = \mathbb{E}^Q[e^{-\int_0^t r_s ds}]$
 - ▶ The risk-neutral (assuming $d\text{rd}\lambda = 0$) survival probability:
 $S_i(0, t) = \mathbb{E}^Q[\mathbf{1}_{\{\tau_i > t\}}] = \mathbf{1}_{\{\tau_i > 0\}} \mathbb{E}^Q[e^{-\int_0^t \lambda_i(s) ds}]$
 - ▶ The forward default rate: $\lambda_i^f(t) = -\frac{\partial \log S_i(0, t)}{\partial t} = \frac{S_i'(0, t)}{S_i(0, t)}$
 - ▶ The weights on the forward rates are given by: $\omega_i(t) = \frac{D(0, t) S_i(0, t)}{\int_0^T D(0, u) S_i(0, u) du}$
- ▶ On a portfolio of CDS each with notional \$1 we pay the average spread:

$$c_{\text{av}}(t) = \sum_{i=1}^n \frac{\mathbf{1}_{\{\tau_i > t\}}}{\sum_{j=1}^n \mathbf{1}_{\{\tau_j > t\}}} \text{CS}_i \quad (2)$$

⇒ The average spread drops when more risky firms default.

Theoretical CDX basis

- ▶ Basket CDS balances payments at *constant* spread on outstanding notional with expected loss:

$$\mathbb{E}^Q \left[\int_0^T e^{-\int_0^t r(u)du} c_{\text{th}} \sum_{i=1}^n \mathbf{1}_{\{\tau_i > t\}} dt \right] = \mathbb{E}^Q \left[\int_0^T e^{-\int_0^t r(u)du} \sum_{i=1}^n \ell_i d\mathbf{1}_{\{\tau_i \leq t\}} \right] \quad (3)$$

- ▶ Well-known result: $c_{\text{th}} = \sum_{i=1}^n \omega_i^n \text{CS}_i$ where:

$$\omega_i^n(t) = \frac{D(0, t) S_i(0, t)}{\sum_{i=1}^n \int_0^T D(0, u) S_i(0, u) du}$$

- ▶ Theoretical basis is *DV01* weighted value of single name CDS spreads.

⇒ Different from average spread on portfolio of CDS.

⇒ Riskier spreads are weighted less.

CDS-CDX basis in practice

- ▶ In practice, even though quoted in spreads, CDX is traded in upfront (u_f) relative to a fixed deal spread (c_{fix}).
- ▶ Upfront calculated from Bloomberg page CDSW (the same used for single name cds):
 - ▶ Extract a constant intensity from spread assuming fixed LGD ($\ell_{\text{fix}} = 0.6$) from :

$$c_{\text{dx}} = \frac{\int_0^T D(0, t) \ell_{\text{fix}} \hat{\lambda} e^{-\hat{\lambda} t} dt}{\int_0^T D(0, t) e^{-\hat{\lambda} t} dt} \equiv \ell_{\text{fix}} \hat{\lambda} \quad (4)$$

- ▶ Compute the fair value of the up-front payment made by the protection buyer:

$$u_f + \int_0^T D(0, t) e^{-\hat{\lambda} t} c_{\text{fix}} n dt = \int_0^T D(0, t) e^{-\hat{\lambda} t} c_{\text{dx}} n dt \quad (5)$$

$$\Rightarrow \boxed{u_f = \int_0^T D(0, t) e^{-\hat{\lambda} t} (c_{\text{dx}} - c_{\text{fix}}) n dt}$$

Exhibit 3.1: The CDSW model on Bloomberg calculates mark-to-market values for CDS contracts

<HELP> for explanation. PL11 Govt CDSW
 1<GO> to save Deal, 2<GO> to save curve source CPU:122

CREDIT DEFAULT SWAP

Deal	Curves	View	Reference Obligation	ISDA Info	Amortization
Deal Information RED Pair:			Spreads D Date		
Reference:			Curve Date:	11/ 9/06	
Counterparty:	Deal#:		Benchmark:	S 23 Ask	
Ticker: /	Series:	Privilege: F Firm	US BGN Swap Curve	Sprds: 0 User Ask TMM	
Business Days: USD	Settlement Code: USD	Currency: USD			
Business Day Adj: 1 Following	Amortizing: N	Knock Out: N			
BUY Notional: 10.00 MM	Day Count: ACT/360	Month End: N			
Effective Date: 11/10/06	Next to Last Cpn: 9/20/11	First Cpn: 12/20/06			
Maturity Date: 12/20/11	Recovery Rate: 0.40	Debt Type: 1 Senior			
Payment Freq: 0 Quarterly	Deal Spread: 425.000 bps				
Pay Accrued: 1 True	Calculator Mode: 1 Calc Price				
Curve Recovery: 1 True	Valuation Date: 11/10/06	Model: J JPMorgan			
Recovery Rate: 0.40	Cash Settled On: 11/14/06				
Deal Spread: 425.000 bps	Price: 97.20473333	Repl Sprd: 500.000 bps			
	Principal: 279,526.67	Days: 0			
	Accrued: 0.00	Sprd DV01: 3,620.56			
	Market Val: 279,526.67	TR DV01: -64.33			

Original Spread → 425.000 bps

Current Spread → 500.000 bps

Mark-to-market value of the position → 279,526.67

The change in price from 1bp change in spread, approx. equal to duration *notional → 3,620.56

Frequency: 0 Quarterly
Day Count: ACT/360
Recovery Rate: 0.40

Australia 61 2 9777 9600 Brazil 55 11 2048 4500 Europe 44 20 7930 7500 Germany 49 69 920410
 Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2006 Bloomberg L.P.
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Exhibit 14.1: CDX CDSW model on Bloomberg

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CREDIT DEFAULT SWAP

Deal	Curve	Mark	Spreads	Date
Deal Information			Spreads	
Reference: DJ CDX 6/11			Curve Date: 5/ 3/06	
Counterparty: CDX.NA.IC.6 Deal: SP9501RK			Benchmark: S 23 Ask	
Ticker: CDX6 CDS Series: 6 5Y			US BGN Swap Curve	
Business Days: USD Settlement Code: USD			Sprds: 0 User Ask	
Business Day Adj: 1 Following Currency: USD			CDS SP9501RK IMM	
B BUY Notional: 10.00 MM Factor: 1			Par Cds Spreads Default	
Effective Date: 3/21/06 Knock Out: N			Flat: Y (bps) Prob	
Maturity Date: 6/20/11 Day Count: ACT/360			12/20/06 34.000 0.0036	
Payment Freq: Q Quarterly Month End: N			6/20/07 34.000 0.0064	
Pay Accrued: T True First Cpn: 6/20/06			6/20/08 34.000 0.0121	
Curve Recovery: T True Next to Last Cpn: 3/21/11			6/22/09 34.000 0.0177	
Recovery Rate: 0.40 Date Gen Method: B Backward			6/21/10 34.000 0.0233	
Deal Spread: 40.000 bps			6/20/11 34.000 0.0289	
Calculator Mode: 1 Calc Price			6/20/13 34.000 0.0399	
Settlement Date: 5/ 4/06 Model: J JPMorgan			6/20/16 34.000 0.0562	
Cash Settled On: 5/ 8/06			Frequency: 0 Quarterly	
Price: 100.26688880 Repl Sprd: 34.000 bps			Day Count: ACT/360	
Market Value: -26,688.88 Days: 44			Recovery Rate: 0.40	
Accrued: -4,888.65 Sprd DV01: 4,464.43				
Total Value: -31,577.77 IR DV01: 6.70				

Index Coupon → Deal Spread: 40.000 bps

Current market spreads → Spreads table (circled)

Upfront Fee = Spread difference * Spread DV01 → Market Value: -26,688.88

Accrual Fee → Accrued: -4,888.65

The change in price from 1bp change in spread, approx. equal to duration * notional → Sprd DV01: 4,464.43

CDX basis in practice

- ▶ Why use this approach?
 - ▶ Advantage of fixed deal spread when netting/assigning/marking to market trades: avoids model-dependent MTM (default intensity and recovery).
 - ▶ Tradition of quoting spreads/ yields instead of price in IG land (\neq HY).

\Rightarrow model dependence transferred to spread \Rightarrow upfront transformation.

\Rightarrow Should affect equilibrium spreads (and potentially derivatives).

- ▶ Spread should solve non-linear equation so that total npv of the payment is 'fair':

$$u_f(c_{\text{fix}}, c_{\text{dx}}) + E^Q \left[\int_0^T e^{-\int_0^t r(u) du} c_{\text{fix}} \sum_{i=1}^n \mathbf{1}_{\{\tau_i > t\}} dt \right] = E^Q \left[\int_0^T e^{-\int_0^t r(u) du} \sum_{i=1}^n \ell_i d\mathbf{1}_{\{\tau_i \leq t\}} \right]$$

Comparing with the theoretical basket spread c_{th} :

$$(c_{\text{dx}} - c_{\text{fix}}) n \int_0^T D(0, t) e^{-\frac{c_{\text{dx}}}{\ell_{\text{fix}}} t} dt = (c_{\text{th}} - c_{\text{fix}}) \int_0^T D(0, t) \sum_{i=1}^n S_i(0, t) dt \quad (6)$$

Quantitative implications

- ▶ $c_{dx} = c_{th} = c_{fix}$ if
 - ▶ Default intensities are constant and identical across all firms,
 - ▶ LGD are constant and identical across all firms and equal to ℓ_{fix} .
- ▶ In practice, neither assumption is satisfied.
- ▶ Using 125 constituents of the IG8, assume intensity is constant (fitted to 5-year single name spreads with fixed recovery $\lambda_i = \ell_{fix}$):

c_{fix}	c_{av}	c_{th}	c_{dx}
35	37.4041	36.9750	36.9748
30	37.4041	36.9750	36.9745

⇒ Impact of heterogeneity in spread is small.

- ▶ Assume intensity is increasing with maturity ($S(t) = e^{-\lambda_t t}$ with $\lambda_t = \lambda_5 e^{-\beta*(5-t)}$):

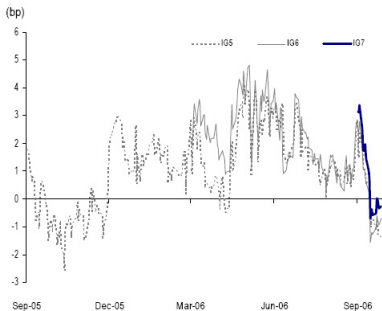
β	c_{fix}	c_{th}	c_{dx}
0	35	36.9750	36.9748
0.2	35	37.0876	37.0961
0.4	35	37.1584	37.1728
0	30	36.9750	36.9745
0.2	30	37.0876	37.1165
0.4	30	37.1584	37.2062

⇒ Impact becomes more noticeable when default term structure is not flat and spread trades away from deal spread.

Quantitative implications

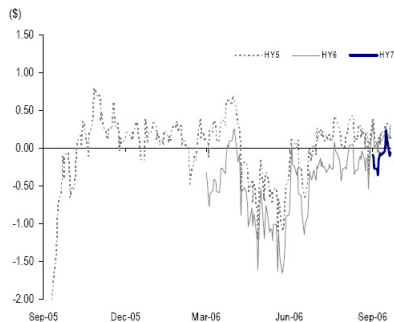
- 'Theoretical basis' in the data:

Exhibit 14.2: CDX IG basis to theoretical tends to be more positive (CDX has wider spread than underlying) in the on-the-run index



Source: JPMorgan

Exhibit 14.3: CDX HY basis to theoretical tends to be more negative (CDX has a lower dollar price than underlying) in the on-the-run index.



- Deal spread readjusted at every roll.

Credit Default Swaptions

- ▶ Credit spread option on CDX gives the call/put holder the right to sell/buy protection at maturity T at a spread-strike of k .
- ▶ Market convention is to settle with payoff equal to mark to market on a CDX position entered at spread of k some previous date:

$$\text{putpayoff} = \max[u_f(\text{cs}) - u_f(k)]$$

- ▶ Implication of the conventions \Rightarrow Discount at two different discount rates.

$$\max[u_f(\text{cs}) - u_f(k), 0] = n \max\left[\int_0^T e^{-rt} \left(e^{-\hat{\lambda}(\text{cs})t} (\text{cs} - c_{\text{fix}}) - e^{-\hat{\lambda}(k)t} (k - c_{\text{fix}}) \right) dt, 0\right]$$

- ▶ Compare with the economically more 'natural' payoff:

$$\text{Op}_2 \equiv \int_0^T e^{-(r+\hat{\lambda}(\text{cs}))t} \max(\text{cs} - k, 0) n dt$$

- ▶ Nothing *wrong*, just a convention. But it should have impact on prices. (analogous to an equity option where we agree that payoff will be Black-Scholes at fixed volatility with a fixed remaining maturity).

Quantitative implications in a Simple Model

- ▶ Assume that all firms are homogeneous with identical LGD and intensity (so $c_{dx} = c_{th}$).
- ▶ Assume intensity process for each firm in basket follows:

$$\frac{d\lambda_t}{\lambda_t} = \begin{cases} \mu dt + \sigma dz(t) & \text{if } 0 \leq t \leq T \\ \lambda_T & \text{if } t > T \end{cases} \quad (7)$$

- ▶ Because of this simplifying assumptions, the basket index CDS rate at maturity will be equal to:

$$c_{dx} = c_{th} = cs(T) = \ell\lambda(T) \quad (8)$$

- ▶ The survival probability is given by:

$$S_T = E^Q \left[e^{-\int_0^T \lambda_t dt} \right] \quad (9)$$

Quantitative Implications

- ▶ Assuming full knock-out and current market conventions, the value of a credit swaption is:

$$\begin{aligned} \text{Op}_1 &= \mathbb{E}^Q \left[e^{-rT} \sum_i \mathbf{1}_{\{\tau_i > T\}} \max \left[\int_T^M \left(e^{-(r+\lambda_T)(t-T)} (\ell\lambda_T - c_{\text{fix}}) - e^{-(r+\frac{k}{\ell})(t-T)} (k - c_{\text{fix}}) \right) dt, 0 \right] \right] \\ &= \sum_{j=0}^n C_n^j (S_T)^{n-j} (1 - S_T)^j e^{-rT} \mathbb{E}^Q \left[\max \left[(\ell\lambda_T - c_{\text{fix}}) H(r + \lambda_T) - (k - c_{\text{fix}}) H\left(r + \frac{k}{\ell}\right), 0 \right] \right] \end{aligned}$$

- ▶ The alternative (more 'natural') option value is:

$$\begin{aligned} \text{Op}_2 &= \mathbb{E}^Q \left[e^{-rT} \sum_i \mathbf{1}_{\{\tau_i > T\}} \int_T^M e^{-(r+\lambda_T)(t-T)} \max(\ell\lambda_T - k, 0) dt \right] \\ &= \sum_{j=0}^n C_n^j (S_T)^{n-j} (1 - S_T)^j (n-j) e^{-rT} \mathbb{E}^Q [\max(\ell\lambda_T - k, 0) H(r + \lambda_T)] \end{aligned}$$

where $H(x) = \frac{1 - e^{-x(M-T)}}{x}$

- ▶ If $k = c_{\text{fix}}$ then $\text{Op}_2 = \text{Op}_1$.

Quantitative implications

- ▶ Set $\sigma = 0.4$, $l_{\text{fix}} = 0.6$, $cs(0) = \lambda(0)$, $l_{\text{fix}} = 37\text{bps}$, $\mu = 0.03$, $r = 0.0535$, $T = 0.5$, and

- ▶ $c_{\text{fix}} = 35\text{bps}$.

	$\frac{Op_1 - Op_2}{Op_2}$		
k	30	35	40
$M=5$	0.0018	0	-0.0018
$M=10$	0.0036	0	-0.0036

- ▶ $c_{\text{fix}} = 45\text{bps}$.

	$\frac{Op_1 - Op_2}{Op_2}$		
k	30	35	40
$M=5$	0.0053	0.0036	0.0018
$M=10$	0.010	0.0072	0.0036

⇒ As $c_{\text{fix}} \neq k$ differences can be more 'sizable' (relatively insensitive to σ, μ).

Venezuela: technical default?

▶ setup

- ▶ Chavez announces he wants to exit the IMF (when?)
- ▶ A few brokers realize that some bonds have a covenant in case Venezuela leaves the IMF. If an investor or a group of investors holds more than 25% of an issue, they can demand accelerated payments of principal. This constitutes a technical default which will trigger all CDS.
- ▶ The spread on Venezuela's CDS widens from around 140 to 200bps (more so in the short maturity one year than in the longer maturity)

▶ What are the consequences of such a technical default?

⇒ CDS Exercise

Protection buyers will have an incentive to deliver discount bonds. Discount bond holders (with the clause - not all have the clause) will have an incentive to demand acceleration. Discount bonds (without the covenant) should richen as they become cheapest to deliver. Protection buyers that only have available premium or par bonds to deliver will choose not to deliver (since they have the option not to deliver and therefore not to exercise their right to protection).

Venezuela: technical default?

⇒ Marking to Market

Many existing positions were marked to market under a standard recovery assumption of 25% and using a default intensity consistent with observed spreads. A technical default with a close to 100% recovery will mean MTM gains or losses that are sometimes surprising. For example, a protection seller with a mark to market loss in his books as a result of a spread widening would see an instantaneous gain as the mark to market suddenly goes to zero.

⇒ Going Forward

- ▶ Short term There are surprising consequences for the trading activity during the period of uncertainty. Dealers refuse to unwind positions as there is substantial uncertainty about the 'correct value' of the market to market of positions (see previous example).
- ▶ Long term There are 'some' possible questions about the future of the sovereign CDS market. The underlying entity is in default but can still issue bonds and, in fact, the existing bonds are all trading at par or at a premium.