"Do (should) Credit Index market conventions have an impact on prices?"

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- The CDS/CDX Market
- The CDX-CDS Basis
- Default Swaptions
- Bonus track: Venezuela case study

Rapid evolution of credit markets

- Innovation in contracts,
  - from traditional *funded* securities: corporate bonds
  - to new *unfunded* derivatives: credit default swaps (CDS)
- And increased liquidity,
- Allow investors to express views on:
  - Single-names CDS
  - Baskets of names (CDX.IG, CDX.HV, iTraxx)
  - Spread volatility (Spread options)
  - Correlation (Synthetic liquid CDO, Bespoke CDO, CDO^2...)
  - Emerging Market Countries (EMCDS)
  - Basket of Countries (EMCDX)

CDS Contract Structure

- A CDS is an insurance contract against a credit event of counterparty:
  - Prior to credit event:

    protection buyer → (notional × spread) → protection seller

  - Upon arrival of credit event:

    protection buyer → (deliverable bond) → protection seller

    protection buyer ← (notional) ← protection seller

- Definition of credit event:

  Bankruptcy
  Failure to pay
  Obligation acceleration or default
  Repudiation/moratorium
  Restructuring (Full R, Mod R, ModMod R, No R)

Arbitrage Relation

- Buy XYZ bond + Buy XYZ protection $\sim$ Earn risk-free rate
- Buy risk-free bond + Sell XYZ protection $\sim$ Earn XYZ bond yield

$$\boxed{\text{CDS spread} \approx Y_{XYZ} - R_f}$$

$\Rightarrow$ CDS allows pure unfunded play on credit risk.

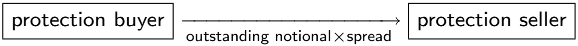
- Empirical evidence on *Basis* = CDS spread $- (Y_{XYZ} - R_f)$.

	Basis wrt Tsy (bp)		Basis wrt Swap (bp)		implied R_f/ Tsy	
	Mean	S.E. (of mean)	Mean	S.E.	Mean	S.E.
Aaa/Aa	-51.30	1.97	9.55	1.31	0.834	0.0250
A	-64.33	1.82	5.83	1.59	0.927	0.0229
Baa	-84.93	3.63	2.21	2.79	0.967	0.0364
All Categories	-62.87	1.38	6.51	1.06	0.904	0.0160

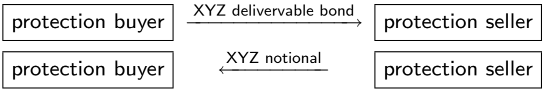
source: Hull, Pedrescu, White (2006)

The CDX index

- The CDX index is an insurance contract against credit events of a portfolio of counterparties (e.g., 125 names in CDX.IG):
  - Prior to credit event:



  - Upon arrival of credit event of XYZ:



  - Following credit event outstanding notional is reduced by notional of XYZ in portfolio (i.e., $\frac{1}{125}$ in CDX.IG).
  - Contract expires at maturity or when notional exhausted.

- Question: What is the (no-arbitrage) relation between the CDX spread and the underlying single-name CDS spreads?

Portfolio of CDS (clearly $\neq$ CDX)

- Single name CDS balances expected coupons and expected loss:

$$\mathrm{E}^Q\left[\int_0^T e^{-\int_0^t r(u)du}\mathrm{cs}_i \mathbf{1}_{\{\tau_i>t\}}\,dt\right] = \mathrm{E}^Q\left[\int_0^T e^{-\int_0^t r(u)du}\ell_i\, d\mathbf{1}_{\{\tau_i\le t\}}\right] \tag{1}$$

- Single-name CDS is weighted average forward losses: $\boxed{\mathrm{cs}_i = \int_0^T \omega_i(t)\lambda_i^f(t)\ell_i\,dt}$

  where:
  - the risk-free zero-coupon bonds: $D(0,t) = \mathrm{E}^Q[e^{-\int_0^t r_s ds}]$
  - The risk-neutral (assuming $drd\lambda = 0$) survival probability: $S_i(0,t) = \mathrm{E}^Q[\mathbf{1}_{\{\tau_i>t\}}] = \mathbf{1}_{\{\tau_i>0\}}\mathrm{E}^Q[e^{-\int_0^t \lambda_i(s)ds}]$
  - The forward default rate: $\lambda_i^f(t) = -\frac{\partial \log S_i(0,t)}{\partial t} = \frac{S_i'(0,t)}{S_i(0,t)}$
  - The weights on the forward rates are given by: $\omega_i(t) = \frac{D(0,t)S_i(0,t)}{\int_0^T D(0,u)S_i(0,u)du}$

- On a portfolio of CDS each with notional \$1 we pay the average spread:

$$\mathrm{c}_{\mathrm{av}}(t) = \sum_{i=1}^n \frac{\mathbf{1}_{\{\tau_i>t\}}}{\sum_{j=1}^n \mathbf{1}_{\{\tau_j>t\}}}\mathrm{cs}_i \tag{2}$$

$\Rightarrow$ The average spread drops when more risky firms default.

Theoretical CDX basis

- Basket CDS balances payments at *constant* spread on outstanding notional with expected loss:

$$\mathrm{E}^Q\left[\int_0^T e^{-\int_0^t r(u)du}\mathrm{c}_{\mathrm{th}}\sum_{i=1}^n \mathbf{1}_{\{\tau_i>t\}}\,dt\right] = \mathrm{E}^Q\left[\int_0^T e^{-\int_0^t r(u)du}\sum_{i=1}^n \ell_i\, d\mathbf{1}_{\{\tau_i\leq t\}}\right] \tag{3}$$

- Well-known result: $\boxed{\mathrm{c}_{\mathrm{th}} = \sum_{i=1}^n \omega_i^n\, \mathrm{cs}_i}$ where:

$$\omega_i^n(t) = \frac{D(0,t)S_i(0,t)}{\sum_{i=1}^n \int_0^T D(0,u)S_i(0,u)du}$$

- Theoretical basis is *DV01* weighted value of single name CDS spreads.

⇒ Different from average spread on portfolio of CDS.

⇒ Riskier spreads are weighted less.

CDS-CDX basis in practice

- In practice, even though quoted in spreads, CDX is traded in upfront (u_f) relative to a fixed deal spread (c_{fix}).

- Upfront calculated from Bloomberg page CDSW (the same used for single name cds):
  - Extract a constant intensity from spread assuming fixed LGD ($\ell_{\text{fix}} = 0.6$) from :

$$\mathrm{c}_{\mathrm{dx}} = \frac{\int_0^T D(0,t)\ell_{\text{fix}}\hat{\lambda}e^{-\hat{\lambda}t}dt}{\int_0^T D(0,t)e^{-\hat{\lambda}t}dt} \equiv \ell_{\text{fix}}\hat{\lambda} \qquad (4)$$

  - Compute the fair value of the up-front payment made by the protection buyer:

$$u_f + \int_0^T D(0,t)e^{-\hat{\lambda}t}c_{\text{fix}}\, n\, dt = \int_0^T D(0,t)e^{-\hat{\lambda}t}\mathrm{c}_{\mathrm{dx}}\, n\, dt \qquad (5)$$

$$\Rightarrow \boxed{u_f = \int_0^T D(0,t)e^{-\hat{\lambda}t}(\mathrm{c}_{\mathrm{dx}} - c_{\text{fix}})\, n\, dt}$$

Exhibit 3.1: The CDSW model on Bloomberg calculates mark-to-market values for CDS contracts

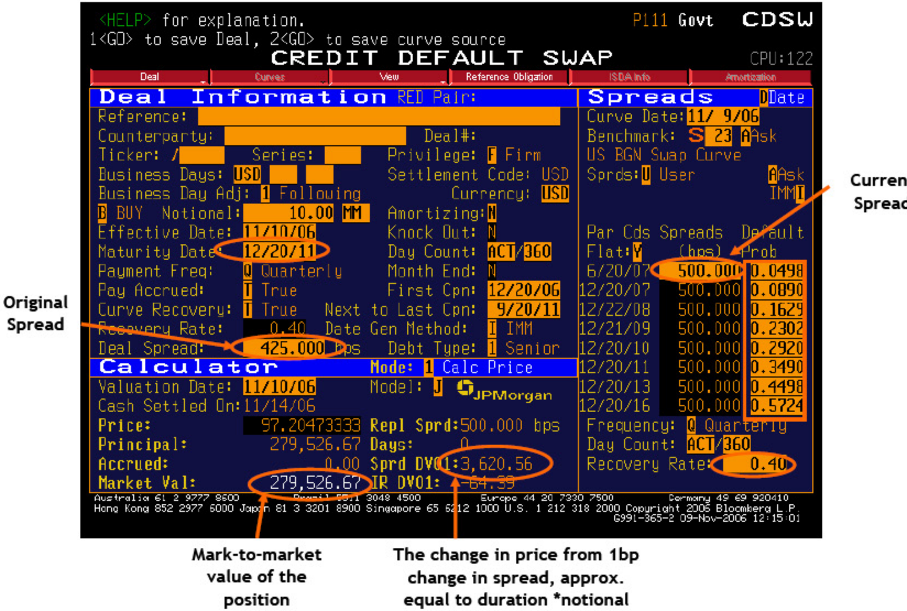
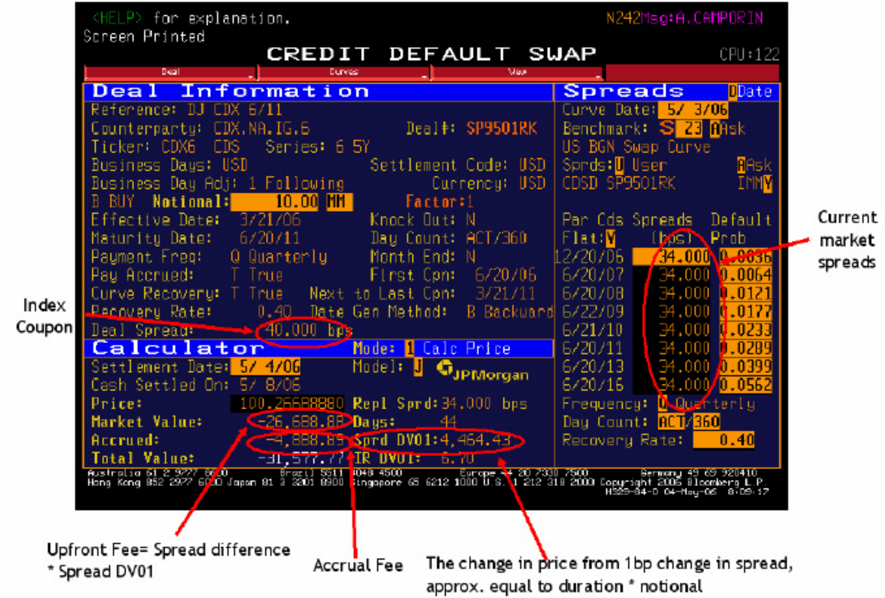


Exhibit 14.1: CDX CDSW model on Bloomberg



Source: Bloomberg

CDX basis in practice

- Why use this approach?
  - Advantage of fixed deal spread when netting/assigning/marking to market trades: avoids model-dependent MTM (default intensity and recovery).
  - Tradition of quoting spreads/ yields instead of price in IG land ($\neq$ HY).

$\Rightarrow$ model dependence transferred to spread$\rightleftharpoons$upfront transformation.

$\Rightarrow$ Should affect equilibrium spreads (and potentially derivatives).

- Spread should solve non-linear equation so that total npv of the payment is 'fair':

$$u_f(c_{\mathrm{fix}}, \mathrm{c}_{\mathrm{dx}}) + \mathrm{E}^Q\left[\int_0^T e^{-\int_0^t r(u)du} c_{\mathrm{fix}} \sum_{i=1}^n \mathbf{1}_{\{\tau_i > t\}} dt\right] = \mathrm{E}^Q\left[\int_0^T e^{-\int_0^t r(u)du} \sum_{i=1}^n \ell_i d\mathbf{1}_{\{\tau_i \leq t\}}\right]$$

Comparing with the theoretical basket spread c_{th}:

$$(\mathrm{c}_{\mathrm{dx}} - c_{\mathrm{fix}})\, n \int_0^T D(0,t) e^{-\frac{\mathrm{c}_{\mathrm{dx}}}{\ell_{\mathrm{fix}}} t} dt = (\mathrm{c}_{\mathrm{th}} - c_{\mathrm{fix}}) \int_0^T D(0,t) \sum_{i=1}^n S_i(0,t)\, dt \qquad (6)$$

Quantitative implications

- $c_{dx} = c_{th} = c_{fix}$ if
  - Default intensities are constant and identical across all firms,
  - LGD are constant and identical across all firms and equal to ℓ_{fix}.

- In practice, neither assumption is satisfied.

- Using 125 constituents of the IG8, assume intensity is constant (fitted to 5-year single name spreads with fixed recovery $\lambda_i = \ell_{fix}$):

c_{fix}	c_{av}	c_{th}	c_{dx}
35	37.4041	36.9750	36.9748
30	37.4041	36.9750	36.9745

$\Rightarrow$ Impact of heterogeneity in spread is small.

- Assume intensity is increasing with maturity ($S(t) = e^{-\lambda_t t}$ with $\lambda_t = \lambda_5 e^{-\beta*(5-t)}$):

β	c_{fix}	c_{th}	c_{dx}
0	35	36.9750	36.9748
0.2	35	37.0876	37.0961
0.4	35	37.1584	37.1728
0	30	36.9750	36.9745
0.2	30	37.0876	37.1165
0.4	30	37.1584	37.2062

$\Rightarrow$ Impact becomes more noticeable when default term structure is not flat and spread trades away from deal spread.

Quantitative implications

- 'Theoretical basis' in the data:

Exhibit 14.2: CDX IG basis to theoretical tends to be more positive (CDX has wider spread than underlying) in the on-the-run index

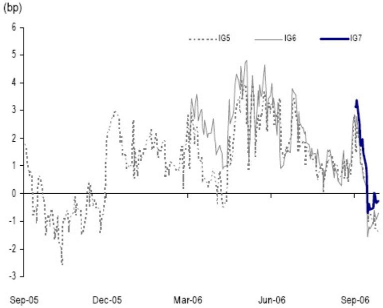
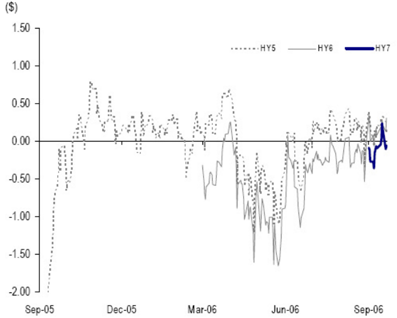


Exhibit 14.3: CDX HY basis to theoretical tends to be more negative (CDX has a lower dollar price than underlying) in the on-the-run index.



Source: JPMorgan

- Deal spread readjusted at every roll.

Credit Default Swaptions

- Credit spread option on CDX gives the call/put holder the right to sell/buy protection at maturity T at a spread-strike of k.
- Market convention is to settle with payoff equal to mark to market on a CDX position entered at spread of k some previous date:

$$\text{putpayoff} = \max[u_f(\mathrm{cs}) - u_f(k)]$$

- Implication of the conventions $\Rightarrow$ Discount at two different discount rates.

$$\max[u_f(\mathrm{cs}) - u_f(k), 0] = n \max[\int_0^T e^{-rt} \left(e^{-\hat{\lambda}(\mathrm{cs})t}(\mathrm{cs} - c_{\mathrm{fix}}) - e^{-\hat{\lambda}(k)t}(k - c_{\mathrm{fix}}) \right) dt, 0]$$

- Compare with the economically more 'natural' payoff:

$$\mathrm{Op}_2 \equiv \int_0^T e^{-(r+\hat{\lambda}(\mathrm{cs}))t} \max(\mathrm{cs} - k, 0)\, n\, dt$$

- Nothing *wrong*, just a convention. But it should have impact on prices. (analogous to an equity option where we agree that payoff will be Black-Scholes at fixed volatility with a fixed remaining maturity).

Quantitative implications in a Simple Model

- Assume that all firms are homogeneous with identical LGD and intensity (so $\mathrm{c}_{\mathrm{dx}} = \mathrm{c}_{\mathrm{th}}$).

- Assume intensity process for each firm in basket follows:

$$\frac{d\lambda_t}{\lambda_t} = \begin{cases} \mu dt + \sigma dz(t) & \text{if} \quad 0 \leq t \leq T \\ \lambda_T & \text{if} \quad t > T \end{cases} \tag{7}$$

- Because of this simplifying assumptions, the basket index CDS rate at maturity will be equal to:

$$\mathrm{c}_{\mathrm{dx}} = \mathrm{c}_{\mathrm{th}} = \mathrm{cs}(T) = \ell\lambda(T) \tag{8}$$

- The survival probability is given by:

$$S_T = \mathrm{E}^Q \left[e^{-\int_0^T \lambda_t dt} \right] \tag{9}$$

Quantitative Implications

- Assuming full knock-out and current market conventions, the value of a credit swaption is:

$$\mathrm{Op}_1 = \mathrm{E}^Q\left[e^{-rT}\sum_i \mathbf{1}_{\{\tau_i>T\}} \max[\int_T^M \left(e^{-(r+\lambda_T)(t-T)}(\ell\lambda_T - c_{\mathrm{fix}}) - e^{-(r+\frac{k}{\ell})(t-T)}(k - c_{\mathrm{fix}})\right) dt, 0]\right]$$

$$= \sum_{j=0}^{n} C_n^j (S_T)^{n-j}(1-S_T)^j e^{-rT}\mathrm{E}^Q\left[\max[(\ell\lambda_T - c_{\mathrm{fix}})H(r+\lambda_T) - (k-c_{\mathrm{fix}})H(r+\frac{k}{\ell}), 0]\right]$$

- The alternative (more 'natural') option value is:

$$\mathrm{Op}_2 = E^Q\left[e^{-rT}\sum_i \mathbf{1}_{\{\tau_i>T\}} \int_T^M e^{-(r+\lambda_T)(t-T)} \max(\ell\lambda_T - k, 0)dt\right]$$

$$= \sum_{j=0}^{n} C_n^j (S_T)^{n-j}(1-S_T)^j (n-j) e^{-rT}\mathrm{E}^Q\left[\max(\ell\lambda_T - k, 0)H(r+\lambda_T)\right]$$

where $H(x) = \frac{1-e^{-x(M-T)}}{x}$

- If $k = c_{\mathrm{fix}}$ then $\mathrm{Op}_2 = \mathrm{Op}_1$.

Quantitative implications

- Set $\sigma = 0.4$, $\ell_{\text{fix}} = 0.6$, $\text{cs}(0) = \lambda(0)$, $\ell_{\text{fix}} = 37bps$, $\mu = 0.03$, $r = 0.0535$, $T = 0.5$, and
  - $c_{\text{fix}} = 35bps$.

$\frac{\text{Op}_1 - \text{Op}_2}{\text{Op}_2}$

k	30	35	40
M=5	0.0018	0	-0.0018
M=10	0.0036	0	-0.0036

  - $c_{\text{fix}} = 45bps$.

$\frac{\text{Op}_1 - \text{Op}_2}{\text{Op}_2}$

k	30	35	40
M=5	0.0053	0.0036	0.0018
M=10	0.010	0.0072	0.0036

$\Rightarrow$ As $c_{\text{fix}} \neq k$ differences can be more 'sizable' (relatively insensitive to σ, μ).

Venezuela: technical default?

- setup
  - Chavez announces he wants to exit the IMF (when?)
  - A few brokers realize that some bonds have a covenant in case Venezuela leaves the IMF. If an investor or a group of investors holds more than 25% of an issue, they can demand accelerated payments of principal. This constitutes a technical default which will trigger all CDS.
  - The spread on Venezuela's CDS widens from around 140 to 200bps (more so in the short maturity one year than in the longer maturity)

- What are the consequences of such a technical default?

⇒ CDS Exercise
Protection buyers will have an incentive to deliver discount bonds. Discount bond holders (with the clause - not all have the clause) will have an incentive to demand acceleration. Discount bonds (without the covenant) should richen as they become cheapest to deliver. Protection buyers that only have available premium or par bonds to deliver will choose not to deliver (since they have the option not to deliver and therefore not to exercise their right to protection).

Venezuela: technical default?

⇒ Marking to Market

Many existing positions were marked to market under a standard recovery assumption of 25% and using a default intensity consistent with observed spreads. A technical default with a close to 100% recovery will mean MTM gains or losses that are sometimes surprising. For example, a protection seller with a mark to market loss in his books as a result of a spread widening would see an instantaneous gain as the mark to market suddenly goes to zero.

⇒ Going Forward

- Short term There are surprising consequences for the trading activity during the period of uncertainty. Dealers refuse to unwind positions as there is substantial uncertainty about the 'correct value'of the market to market of positions (see previous example).
- Long term
  There are 'some' possible questions about the future of the sovereign CDS market. The underlying entity is in default but can still issue bonds and, in fact, the existing bonds are all trading at par or at a premium.