1. Consider a field theory with Weyl fermion fields (ψ, ψ_c, λ) with electric charges respectively equal to $(1. - 1, 0)$, with scalars (ϕ, ϕ_c) with charges equal to $(1, -1)$ and lagrangian

$$\mathcal{L} = \mathcal{L}_{kin} + \left[g_1 \psi \lambda \phi_c + g_2 \psi_c \lambda \phi + \frac{m}{2} \lambda\lambda + m_3^2 \phi \phi_c + \text{h.c.} \right] \quad (0.1)$$

$$+ \quad m_1^2 |\phi|^2 + m_2^2 |\phi_c|^2 + (g_3 (\phi \phi_c)^2 + \text{h.c.}) + g_4 |\phi|^4 + g_5 |\phi_c|^4 + g_6 |\phi \phi_c|^2 \quad (0.2)$$

   Is the fermion ψ naturally massless in such a theory? Explain the result using symmetries. What about the case $m_3 = 0$. (Assume the scalar potential is positive definite so that no scalar acquires a vacuum expectation value).

2. Consider now adding a neutral scalar η to the above model, modifying the lagrangian to

$$\mathcal{L} = \mathcal{L}_{kin} + [g_1 \psi \lambda \phi_c + g_2 \psi_c \lambda \phi + g_7 \eta \lambda\lambda + m_3 \eta^* \phi \phi_c + \text{h.c.}] \quad (0.3)$$

$$+ \quad m_1^2 |\phi|^2 + m_2^2 |\phi_c|^2 + m_3^2 |\eta|^2 + g_4 |\phi|^4 + g_5 |\phi_c|^4 + g_6 |\phi \phi_c|^2 \quad (0.4)$$

   Are fermions naturally massless in the above field theory? Explain the result as before (again the scalar potential is assumed positive definite).